GADSDEN TECHNICAL INSTITUTE
CONTINUAL EDUCATION
COVID-19 EMERGENCY LESSONS

Teacher Name: Mr. Alfred Suber
Dates of Instruction: March 30 – April 13, 2020
Lesson Title: Introduction to Construction Math
Grade Levels: 10 – 12; Adult
Subject Area: Carpentry

Assignment: After reading the material on introduction to construction math, the student will be able to demonstrate mathematics knowledge and skills relevant to the carpentry field and the student will be able to: apply geometry and algebra skills to solve math problems related to carpentry with and without a calculator; demonstrate knowledge of arithmetic operations; solve problems for distance, perimeter, area and volume; analyze and apply data and measurements to solve problems and interpret documents; construct charts/graphs using functions and data.

Lesson Instructions:
Week of March 30 – April 3, 2020, read sections 1, 2 and 3 pages 1-28. Study and Learn Trade Terms Definitions on pages 70-71.

Week of April 6 – 15, 2020, read sections 4, 5, and 6 pages 30-57. Study and Learn Trade Terms Definitions on pages 70-71.

Practice Activities:
Week of March 30 – April 3, 2020, answer section review questions on pages 8, 16 and 29.

Week of April 6 – 15, 2020, answer section review questions on pages 36, 47 and 57. Answer review questions pages 58, 59, and 60. Answer trade terms quiz questions on pages 61-62.

Instructional Materials:
1. Carpentry Introduction to Introduction to Construction Math Module 2 reading packet
2. Carpentry Introduction to Introduction to Construction Math Module 2 questions packet.

Special Notes from Instructor:
ALL paper work should be kept in your folder, signed and dated to reflect completion date(s) prior to bringing them to class with you on April 16, 2020. If there are any questions, I can be reached at (850) 875-8324; ext. 5121 or email suberj@gcpsmail.com.

SECTION ONE

1.0.0 WHOLE NUMBERS

Objective

Identify whole numbers and demonstrate how to work with them mathematically.

a. Identify different whole numbers and their place values.
b. Demonstrate the ability to add and subtract whole numbers.
c. Demonstrate the ability to multiply and divide whole numbers.

Trade Terms

Decimal: A part of a number represented by digits to the right of a point, called a decimal point. For example, in the number 1.25, .25 is the decimal portion of the number. In this case, it represents 25 percent of the whole number 1.

Difference: The result of subtracting one number from another. For example, in the problem 8 − 3 = 5, 5 is the difference between the two numbers.

Digit: Any of the numerical symbols 0 to 9.

Dividend: In a division problem, the number being divided is the dividend.

Divisor: In a division problem, the number that is divided into another number is called the divisor.

Equation: A mathematical statement that indicates that the value of two mathematical expressions, such as 2 × 2 and 1 × 4, are equal. An equation is written using the equal sign in this manner: 2 × 2 = 1 × 4.

Fraction: A portion of a whole number represented by two numbers. The upper number of a fraction is known as the numerator and the bottom number is known as the denominator.

Negative numbers: Numbers less than zero. For example, −1, −2, and −3 are negative numbers.

Place value: The exact value a digit represents in a whole number, determined by its place within the whole number or by its position relative to the decimal point. In the number 124, the number 2 represents 20, since it is in the tens position.

Positive numbers: Numbers greater than zero. For example, 1, 2, and 3 are positive numbers. Any number without a negative (−) sign in front of it is considered to be a positive number.

Product: The answer to a multiplication problem. For example, the product of 6 × 6 is 36.

Quotient: The result of a division problem. For example, when dividing 6 by 2, the quotient is 3.

Remainder: The amount left over in a division problem. For example, in the problem 34 ÷ 8, 8 goes into 34 four times (8 × 4 = 32) with 2 left over; in other words, 2 is the remainder.

Sum: The resulting total in an addition problem. For example, in the problem 7 + 8 = 15, 15 is the sum.

Whole numbers: Complete number units without fractions or decimals.

No matter which construction trade you enter, you can be sure that you will be required to work with **whole numbers** in both your written and oral communication. The ability to read, write, and communicate whole numbers to others accurately is extremely important on any job site. Carpenters will often use whole numbers during the early phases of planning to quickly estimate the square feet of drywall, or linear feet of baseboard necessary to finish a room. Sheet metal workers will use whole numbers during the planning phase for installing an air handling system and to estimate, with relative accuracy, trunk line dimensions, lengths, and the amount of air it will be required to handle.

Be aware that a mistake made when communicating any number to others can result in wasted time, effort, and materials, all of which negatively affect the bottom line.

In this section, you will learn how to work with whole numbers. Whole numbers are complete number units without **fractions** or **decimals**. This section presents whole numbers only. Working with fractions and decimals will be covered in other sections.

Did You Know?

Whole Numbers

The following are whole numbers...
1 5 67 335 2,654

... but the following are *not* whole numbers:
½ ¾ 7⅛ 0.45 4.25

1.1.0 Place Values of Whole Numbers

Each **digit** that makes up a given whole number has a specific **place value**. A digit is any of the numerical symbols from 0 to 9. *Figure 1* provides an example of a whole number with seven digits. To read this seven-digit whole number out loud, say "five million, three hundred sixteen thousand, two hundred forty-seven."

Each of this whole number's seven digits represents a specific place value. Each digit has a value that depends on its place, or location, in the whole number. In this whole number, for example, the place value of the 5 is five million, while the place value of the 2 is two hundred.

Other important points to keep in mind about whole numbers include the following:

- Numbers larger than zero are called **positive numbers** (such as 1, 2, 3 …). Except for zero, all numbers without a minus sign in front of them are positive.
- Numbers less than zero are called **negative numbers** (such as –1, –2, –3 …). Negative numbers are preceded by a negative (–) sign. A number does not need to be positive to still be considered a whole number; negative numbers can also be whole numbers.
- Zero (0) is neither positive nor negative.

Some whole numbers may contain the digit zero. For example, the whole number 7,093 has a zero in the hundreds place. When you read that number out loud, you would say "seven thousand ninety-three."

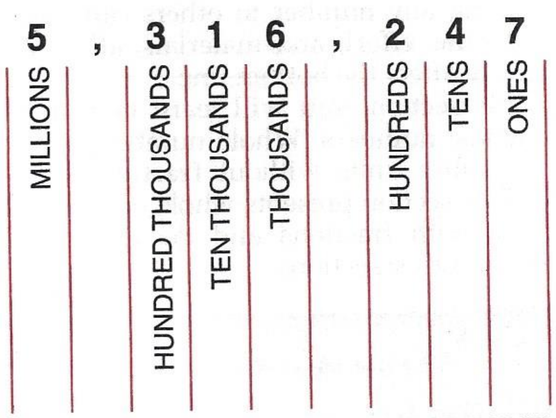

Figure 1 Place values.

1.1.1 Study Problems: Place Values of Whole Numbers

1. Look at the following description of a number. This number would be written as _____.
   Digit in the hundreds place: 9
   Digit in the ones place: 4
   Digit in the thousands place: 3
   Digit in the tens place: 6
   a. 3,964
   b. 4,693
   c. 30,964
   d. 39,064
2. In the number 25,718, the numeral 5 is in the _____.
   a. tens place
   b. thousands place
   c. ones place
   d. hundreds place
3. An estimate for a commercial flooring job requires one thousand, six hundred ninety-three square meters of carpet to complete the first floor. How would you write this amount as a whole number?
   a. 163
   b. 1,693
   c. 10,693
   d. 16,093
4. A supervisor estimates that a commercial building will require sixteen thousand, five hundred feet of copper piping to complete all of the restroom facilities. How would you write this value as a whole number?
   a. 1,650
   b. 16,500
   c. 160,500
   d. 16,000,500
5. An engineer estimates that the total cost to install a building's HVAC system will be three hundred twenty-two thousand, nine hundred and seven dollars. How would you write this cost using digits?
   a. $3,297.00
   b. $300,297.00
   c. $322,907.00
   d. $322,000,907.00

1.2.0 Adding and Subtracting Whole Numbers

To add means to combine the values of two or more numbers together into one **sum** or total. To add whole numbers, use the following steps:

Step 1 Line up the digits in the top number and the bottom number by place value columns.

$$\begin{array}{r} 723 \\ \underline{+\ 84} \end{array}$$

Step 2 Beginning at the right side, add the numbers in the ones column (3 and the 4) together first.

$$\begin{array}{r} 723 \\ \underline{+\ 84} \\ 7 \end{array}$$

Step 3 Continue to add the digits in each column, moving from right to left, one column at a time. In this example, when you add the 2 and the 8 in the tens column you get 10. This requires you to carry the 1 in the tens column over to the next column to the left. To do so, place the 0 in the tens column and carry the 1 over to the top of the hundreds column as shown. That carried-over number is now added to the rest of the digits in that column.

$$\begin{array}{r} \overset{1}{7}23 \\ \underline{+\ 84} \\ 07 \end{array}$$

Step 4 Add the 7 already in the hundreds column to the 1 carried over. The resulting sum is 807.

$$\begin{array}{r} \overset{1}{7}23 \\ \underline{+\ 84} \\ 807 \end{array}$$

Note that you may need to carry-over digits several times in the same problem. The following is an example of such a problem:

$$\begin{array}{r} \overset{1}{6}\overset{1}{6},\overset{1}{7}23 \\ \underline{+\ 5,784} \\ 72,507 \end{array}$$

To subtract means to take away a given amount of one number from the total amount of a second number to find the **difference**. To subtract whole numbers, use the following steps:

Step 1 Line up the digits in the top number and the bottom number by place value columns. Generally, position the larger number on the top. If not, the result will be a negative number. That is appropriate for some calculations, but it is extremely rare in math used on the job site.

$$\begin{array}{r} 12,766 \\ \underline{-\ 1,483} \end{array}$$

Step 2 As in addition, start with the right column—the ones column. Subtract the 3 from the 6 to get 3, and record it under the ones column. As you work your way left into the tens column, note that you are unable to subtract 8 from 6. This will require you to borrow a 1 from the hundreds column. To borrow, cross out the 7 in the hundreds column, change it to a 6, and carry a 1 over to the 6 in the tens column. This makes it 16 instead of 6. Subtract 8 from 16 to get 8, and record it in the tens column.

$$\begin{array}{r} 12,\overset{6}{\cancel{7}}\overset{1}{6}6 \\ \underline{-1,483} \\ 83 \end{array}$$

Step 3 Continue to work towards the left, column by column, completing each subtraction for the remaining place values.

$$\begin{array}{r} 12,\overset{6}{\cancel{7}}\overset{1}{6}6 \\ \underline{-1,483} \\ 11,283 \end{array}$$

Note that you may need to borrow several times in one problem to calculate the difference. The following problem provides an example of this situation:

$$\begin{array}{r} 9\overset{3}{\cancel{4}}\overset{12}{\cancel{3}},\overset{10}{\cancel{1}}\overset{15}{\cancel{5}}3 \\ \underline{-436,372} \\ 506,781 \end{array}$$

1.2.1 Study Problems: Adding and Subtracting Whole Numbers

Use addition and subtraction to solve the following problems. Read each question carefully to determine the appropriate procedure. Be sure to show all of your work.

1. In calculating a bid for a roof restoration, a contractor estimates that he will need $847 for lumber, $456 for roofing shingles, and $169 for hardware. What is the total cost for the materials portion of the bid?

   $ ________

2. Brazil's currency is called the real, and is denoted with this symbol: R$. A plumbing contractor allotted R$10,236 in his bank account to complete three residential jobs. If he estimates Job 1 to cost R$2,477, Job 2 to cost R$2,263, and Job 3 to cost R$3,218, how much money will he have left in the account for unexpected costs?

   R$ ________

3. An HVAC contracting company sent out three work crews to complete three installations over the past week. If Crew One worked 10-, 9-, 11-, 12-, and 9-hour days, Crew Two worked 9-, 12-, 12-, 9-, and 9-hour days, and Crew Three worked 12-, 12-, 10-, 9-, and 11-hour days, how many total hours did the three crews work for the week?

   ________ hours

4. A general contractor ordered three different sized windows to complete a job on a residential home. She ordered a bow window that cost $874; one 36" × 36" double-hung window that cost $67; and one 36" × 54" double-hung window that cost $93. If she had set aside $1,250 to purchase the windows in her estimate, how much will she have left after buying them?

   $ ________

5. Russia's currency is called the ruble, and is denoted with this symbol: Rub. An electrical contractor has Rub850,360 in his business's bank account. If at the end of the week he deposits Rub119,980 in payments made from clients and then pays out Rub79,205 in wages, how much money will he have left in his account?

   Rub ________

Did You Know?

Numeral Systems

People in ancient Egypt, Babylon, Greece, and Rome developed different numeral systems or ways of writing numbers. Some of these early systems were very complex and difficult to use. A new numeral system came into use around 750 BC. Originally developed by the Hindus in India, the system was spread by Arab traders. The Hindu-Arabic system uses only ten symbols—1, 2, 3, 4, 5, 6, 7, 8, 9, 0—and is still in use today. These ten symbols (also called numerals or digits) can be combined to write any number.

1.3.0 Multiplying and Dividing Whole Numbers

Multiplication is a quick way to add the same number to itself numerous times. For example, assume four different people each gave you $8 to pick up lunch for the crew. You could add $8 together four times. However, it is easier to multiply 4 times 8 to get a **product** of 32 than it is to add 8 together four times. With larger numbers, it could take a long time to complete all the addition. To multiply whole numbers, use the following steps:

Step 1 Line up the digits in the top number and the bottom number by place value columns. Position the greater number on the top. This is a matter of convenience and makes the task easier. However, note that it will have no effect at all on the accuracy of the result.

$$\begin{array}{r} 374 \\ \times\ 26 \\ \hline \end{array}$$

Step 2 Start with the ones column (the right column) and multiply the 6 by the 4 (6 × 4 = 24). Write down the 4 in the ones column and carry the 2 up to the tens column. Unlike addition, you will multiply the 6 by every digit in the top number before proceeding to the 2.

$$\begin{array}{r} {\scriptstyle 2} \\ 374 \\ \times\ 26 \\ \hline 4 \end{array}$$

Step 3 Multiply the 6 by the 7 next, and then add the 2 that was carried over ($6 \times 7 = 42 + 2 = 44$). Write down the 4 in the tens column and carry the 4 to the hundreds column.

$$\begin{array}{r} {\scriptstyle 4\,2} \\ 374 \\ \times 26 \\ \hline 44 \end{array}$$

Step 4 Next multiply the 6 by the 3, and then add the 4 that you previously carried over ($6 \times 3 = 18 + 4 = 22$). Write down a 2 in the hundreds column and a 2 in the thousands column, since there are no more places in the upper number.

$$\begin{array}{r} {\scriptstyle 4\,2} \\ 374 \\ \times 26 \\ \hline 2{,}244 \end{array}$$

Step 5 You will then need to multiply each digit in 374 by the 2 in 26. Begin by multiplying by the 2 by the four in the ones column ($2 \times 4 = 8$). Write the 8 down under the tens column, directly under the 2. Notice that you do not write the 8 down in the ones column because the 2 in the number 26 is in the tens column. A zero (0) can be placed in the ones column to help keep the columns aligned if it helps.

$$\begin{array}{r} 374 \\ \times 26 \\ \hline 2{,}244 \\ 80 \end{array}$$

Step 6 Now multiply the 2 by the 7 in the tens column ($2 \times 7 = 14$). Write down a 4 in the hundreds column and carry the 1.

$$\begin{array}{r} {\scriptstyle 1} \\ 374 \\ \times 26 \\ \hline 2{,}244 \\ 480 \end{array}$$

Step 7 Multiply the 2 by the 3, and then add the 1 that you carried over ($2 \times 3 = 6 + 1 = 7$). Write a 7 in the thousands column.

$$\begin{array}{r} {\scriptstyle 1} \\ 374 \\ \times 26 \\ \hline 2{,}244 \\ +7{,}480 \\ \hline \end{array}$$

Step 8 The next step requires addition. Add the two products to get a final product of 9,724. There will be as many products to add together as there are numbers in the bottom portion of the problem. In this example, there are two numbers in the bottom portion of the problem (the 2 and the 6). Thus, there are two products to add together to arrive at the final product.

$$\begin{array}{rl} {\scriptstyle 1} & \\ 374 & \\ \times 26 & \\ \hline 2{,}244 & \text{product of } 6 \times 374 \\ +7{,}480 & \text{product of } 20 \times 374 \\ \hline 9{,}724 & \text{final product} \end{array}$$

Division is the opposite of multiplication. Instead of adding a number several times ($5 + 5 + 5 = 15$, or $5 \times 3 = 15$), you subtract a number several times to find a **quotient** in division. The two numbers in a division problem have their own names. The number being divided is known as the **dividend**. The number the dividend is being divided by is called the **divisor**.

When adding, subtracting, or multiplying whole numbers, the result of the work always results in a whole number. However, in division, a given number may not be neatly divisible by another given number. In this case, the result will be a whole number and/or part of a number. For example, dividing 6 by 3 results in the whole number 2. But what happens when we divide 6 by 4? The four can only be divided into the 6 one time, with 2 left over. This left over portion is referred to as the **remainder**.

Construction Estimating

Creating accurate estimates is an essential part of the job for most contractors. A job that is engineered typically has a budget for the project that the engineer has to work within. For example, granite cannot be specified for a countertop unless the project owner has the money in the budget. Once a design is completed, various contractors can then estimate the cost of the construction, including materials, labor, and the overhead—costs that are associated with putting the labor on the job, keeping the lights and telephones on, and paying the accounting department for the work they do. Then the desired amount of profit is added. The job usually goes to the lowest bidder.

Long division, as will be demonstrated here, requires a sequence of mathematical operations. Not only is division required, but multiplication and subtraction are also required repeatedly.

To divide whole numbers, use the following steps:

Step 1 Begin by setting up the division problem using the division bar as shown below. The ÷ symbol can also be used, writing the problem as 2,638 ÷ 24. However, the division bar provides much better organization when working a problem with pencil and paper. Position the divisor on the left side of the division bar and place the dividend on the right side of the division bar. In this example, 24 is the divisor and 2,638 is the dividend.

```
24 ) 2,638
```

Step 2 Unlike addition, subtraction, and multiplication, where you start the procedure in the ones column, with division you start with the place value of the dividend farthest to the left. In this example, you would first try to divide 24 into 2. However, 2 cannot be divided by 24 at least one full time. As a result, the first two digits must be considered together. You then divide 24 into the 26 of 2,638. If, for example, the divisor was 124 instead of 24, you would need to work with the first three digits. 24 does go into 26 one full time, so write a 1 above the 6 and write 24 under the 26 in 2,638. The 24 is brought down because it is the result of 1 × 24. Then subtract 24 from 26 to get 2. Remember to use zeros (0) to help keep your place values straight.

```
      0,1??
24 ) 2,638
    - 24
      02
```

Step 3 Bring down the next number in 2,638 (the 3) and place it next to the 2. Next, determine if 24 can go into 23. The answer is no, so place a 0 above the 3 on the answer line. 24 times 0 is 0, so write zeros in the appropriate place value columns and subtract the two numbers to get 23.

```
      0,10?
24 ) 2,638
    - 24
      023
    - 000
      023
```

Step 4 Bring down the next number in 2,638 (the 8) and place it next to the 3. Then determine if 24 can go into 238. Yes it can, but how many times? This is where the value of learning multiplication tables can be very valuable. To figure this, think about numbers that are easy to work with. 10 times 24 is 240. That is almost the same as 238, but just a little over. Therefore, you quickly see that 24 will go into 238 nine times. Write a 9 in the answer line next to the 0. Write 216 below the 238 and subtract to get a remainder of 22. So the final answer is that 24 will go into 2,638 one hundred nine times with a remainder of 22. The answer is written as 109 r22, with 109 being the quotient for the problem.

```
    010?           0109 r22
24)2,638       24)2,638
  -24             -24
    023             023
   -000            -000
    0238            0238
                   -0216
                      22
```

Numerators and Denominators

Working with fractions will also be presented in this module. Think of a fraction as just another way to write a division problem. The number on top of a fraction is known as the numerator, but it is the same thing as the dividend in a long division problem. The lower number, called the denominator, is the divisor. The biggest difference in fractions and long division is that most fractions you will encounter on the job site result in a quotient that is less than one. In the average long division problem, the quotient is usually greater than one. But, in the end, fractions simply represent another way to write a division problem. In the case of many fractions you will encounter, the quotient of the division problem is not important—the fraction itself provides the needed information.

Refer back to Step 4. Note the discussion about how many times the number 24 can be divided into 238. The following fact is important to remember in long division problems like these: the answer to each division step should always be a number from 0-9. If the answer is 10 or more, then something went wrong. Either a multiplication error has been made, or 24 could actually be divided into the digits (in this case, the 23 of 238) at least one full time after all.

1.3.1 The Order of Operations

Many math problems cannot be solved without using more than one operation. Problems with multiple operations are most often found on one or both sides of an **equation**. Although no complex equations are presented in this mod, even the simplest equations must be completed using a specific order of operations.

In order for an answer to be accurate, operations must be done in the proper order. The order of operations for math was actually established in the 1500s. For simple equations, the order is: multiplication, division, addition, and subtraction. A simple acronym—MDAS—can remind you of this order. To remember this acronym, think of it as "My Dear Aunt Sally."

The following equation is a good example:

$$6 + 3 \times 5 = A$$

Adding 6 and 3, and then multiplying the sum by 5 results in an answer of 45. However, multiplying 3 times 5 first, and then adding 6 results in an answer of 21. Following the correct order of operations shows that 21 is, in fact, the correct answer. As you can see, following the proper order makes a big difference in the result.

A second acronym—PEMDAS— can be used for slightly more complicated equations. The P represents parentheses, and the E represents exponents. Therefore, if an equation has an operation in parentheses, such as (6 × 3), that operation is done first. A number with an exponent, such as 6^2, would be completed next. Some remember this acronym through the phrase, "Please Excuse My Dear Aunt Sally."

Did You Know?

Mathmatical Nomenclature

Except for the division bar (which actually has no official name but has been referred to as the obelus), every other part of a math problem has an official name.

Addition	3 addend + 4 addend 7 sum
Subtraction	5 minuend − 2 subtrahend 3 difference
Multiplication	4 multiplicand × 3 multiplier 12 product
Division	3 quotient divisor 5)15 dividend

1.3.2 Study Problems: Multiplying and Dividing Whole Numbers

Use multiplication and division to solve the following practical problems. Read each question carefully to determine the appropriate procedure. Be aware that addition or subtraction may also be required. Be sure to show all of your work.

1. Your supervisor sends you to the truck for 180 special fasteners. When you get there, you find that the fasteners come in bags of 15. How many bags of fasteners will you need to bring back?

   ________ bags

2. If the following amounts of lumber need to be delivered to each of 2 different staging areas at 4 different job sites, how many total boards of each size will you need?

   a. (65) 2 × 4s ________

   b. (45) 2 × 8s ________

   c. (25) 2 × 10s ________

3. If one plumbing job requires 45 meters of PVC pipe, and a second job requires 30 meters, how many lengths of pipe will you need if it comes in 6-meter lengths? Remember that you cannot order a partial length of pipe; only orders for whole lengths are generally accepted.

   ________ lengths of pipe

   How much pipe will be left over, assuming there are no errors?

   ________ meters

4. If a crane rental company charges $800 per day, $3,400 per week (5 days), and $10,500 per month:

   a. How much would it cost to rent the crane for 3 days? ________

   b. How much would it cost to rent the crane for 12 days? ________

   c. How much would it cost for one month and 11 days? ________

Additional Resources

Applied Construction Math: A Novel Approach, NCCER. 2006. Upper Saddle River, NJ: Prentice Hall.

Mathematics for Carpentry and the Construction Trades, Alfred P. Webster; Kathryn B. Judy. 2001. Upper Saddle River, NJ: Prentice Hall.

Mathematics for the Trades: A Guided Approach, Robert A. Carman; Hal Saunders. 2014. Pearson Learning.

1.0.0 Section Review

1. Given the number 92475, which digit is in the ten-thousands column of place value?
   a. 2
   b. 4
   c. 7
   d. 9

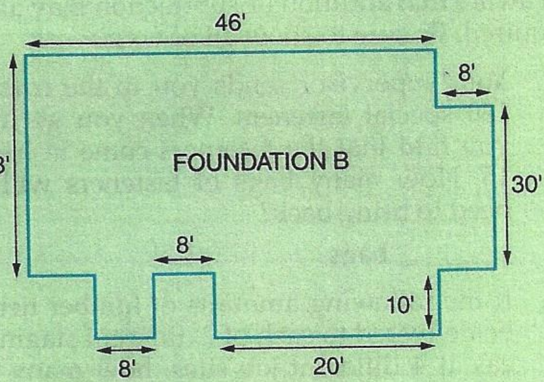

Figure 1 00102-15_SR01.EPS

2. What is the distance around the foundation shown in Section Review Question *Figure 1*?
   a. 206 feet
   b. 214 feet
   c. 224 feet
   d. 234 feet

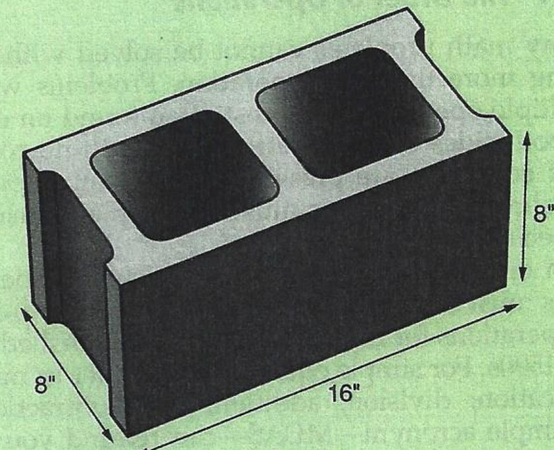

Figure 2 00102-15_SR02.EPS

3. How many concrete blocks like the one shown in Section Review Question *Figure 2* are required to erect a 2-foot high wall for the foundation shown in *Figure 1*? Assume there is no waste, and round your answer *up* to the nearest whole number (since only whole blocks can be purchased).
   a. 336 blocks
   b. 504 blocks
   c. 514 blocks
   d. 528 blocks

SECTION TWO

2.0.0 FRACTIONS

Objective

Explain how to work with fractions.

a. Define equivalent fractions and show how to find lowest common denominators.
b. Describe improper fractions and demonstrate how to change an improper fraction to a mixed number.
c. Demonstrate the ability to add and subtract fractions.
d. Demonstrate the ability to multiply and divide fractions.

Trade Terms

Denominator: The part of a fraction below the dividing line. For example, the 2 in 1/2 is the denominator. It is equivalent to the divisor in a long division problem.

Equivalent fractions: Fractions having different numerators and denominators but still have equal values, such as the two fractions 1/2 and 2/4.

Improper fraction: A fraction whose numerator is larger than its denominator. For example, 5/4 and 6/3 are improper fractions.

Invert: To reverse the order or position of numbers. In fractions, inverting means to reverse the positions of the numerator and denominator, such that 3/4 becomes 4/3. When you are dividing by fractions, one fraction is inverted.

Mixed number: A combination of a whole number with a fraction or decimal. Examples of mixed numbers are 3 7/16, 5.75, and 1 1/4.

Numerator: The part of a fraction above the dividing line. For example, the 1 in 1/2 is the numerator. It is the equivalent of the dividend in a long division problem.

A fraction divides whole numbers into parts. Common fractions are written as two numbers, separated by a slash or by a horizontal line, like this: 1/2

The slash or horizontal line means the same thing as the ÷ sign. So think of a fraction as another way to write a division problem. The fraction 1/2 means 1 divided by 2, or one divided into two equal parts. This fraction is spoken as one-half.

The lower number of the fraction, known as the **denominator**, tells you the number of parts by which the upper number is being divided. In a typical division problem, it is the divisor. The upper number, known as the **numerator**, is a whole number that tells you how many parts are going to be divided. It is the dividend of this division problem. In the fraction 1/2, the 1 is the upper number, or numerator, and the 2 is the lower number, or denominator. A fraction in which both the numerator and the denominator are the same number (2/2, 8/8, 16/16) is always equal to the number 1.

2.1.0 Equivalent Fractions and Lowest Common Denominators

Equivalent fractions, such as 1/2, 2/4, and 4/8, have the same value. *Figure 2* shows the individual units of 1/2, 1/4, and 1/8. It is often easier and more practical to work with fractions that have been reduced to their lowest common denominator. For the three equivalent fractions shown above, 1/2 represents the fraction with the lowest common denominator. Equivalent fractions and finding the lowest common denominator are presented in this section.

2.1.1 Finding Equivalent Fractions

If you are used to working with fractions, it is instantly apparent that 1/2, 2/4, and 4/8 are equivalent fractions. However, there is a mathematical way to determine this. Each fraction's value can be determined by dividing the denominator by the numerator. So 2 divided by 1 equals 2; 4 divided by 2 equals 2; and 8 divided by 4 also equals 2. Since the quotient for all three of these is the same (2), they can be proven to be equivalent fractions. It can be proven physically as well. If pieces of wood are cut at 1/2-inch long, 2/4-inch long, and 4/8-inch long, all three pieces of wood are the same length.

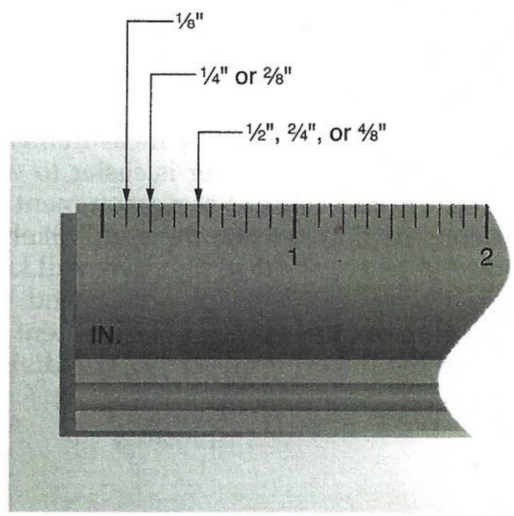

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Figure 2 Equivalent fractions.

Around the World

Fractions on the Job

Those of you who use the inch-pound system of measurement will realize how important it is to understand fractions from the first day you report to work on a construction site. In the real world, most measurements are not whole numbers. Typically, pipe, lumber, and other materials will be measured and cut to fractional lengths such as ⅜, 5⁄16, or ¾ of an inch or foot. Being comfortable working with fractions is an essential job skill for all, but especially for users of the inch-pound system. The metric system of measurement eliminates the need for fractions in day-to-day measurements.

When you measure objects, it is often best to record all measurements as the same fractions—in sixteenths of an inch, for example. Technically, this means using the same denominator for each measurement. Doing this allows you to easily compare, add, and subtract fractional measurements.

Suppose that you are taking measurements and want all of them to be recorded in sixteenths of an inch. To find out how many sixteenths of an inch are equal to ½ inch, for example, you need to multiply both the numerator and the denominator by the same number. As long as the numerator and denominator of a given fraction are being multiplied by the same number, you are creating an equivalent fraction.

For this example, ask yourself what number you would multiply by 2 to get 16. The answer is 8, so you multiply both numbers (the numerator and the denominator) by 8.

$$\frac{1}{2} \times \frac{8}{8} = \frac{8}{16}$$

The answer is 8⁄16 inch, equivalent to ½ inch.

2.1.2 Reducing Fractions to Their Lowest Terms

If you find that the measurement of something is 4⁄16", you may want to reduce the measurement to its lowest terms so the number is easier to work with. In fact, if you call out a measurement like 4⁄16" on a construction job site, there will likely be some puzzled listeners. Although they will know what you mean, they will not understand why you chose to communicate the measurement that way. To find the lowest terms of 4⁄16", use division as follows:

Step 1 To reduce a fraction, determine the largest number that you can divide evenly into both the numerator and the denominator. With a little practice, this is a mental exercise that should not require pencil and paper. If there is no number (other than 1) that will divide evenly into both numbers, the fraction is already in its lowest term.

Step 2 Divide the numerator and the denominator by the number determined in Step 1; both must be divided by the same number. In this example, divide both the numerator and the denominator by 4.

$$\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$$

This shows that the lowest term of 4⁄16 is ¼. Again, the number divided into the numerator and denominator must be the same number, and it must divide into both numbers evenly. In other words, a whole number must result from both operations.

2.1.3 Comparing Fractions and Finding Lowest Common Denominators

Which measurement is larger: ¾" or ⅝"?

This question may be best answered this way: Would you have a longer piece of lumber if you had three sections from a board that was cut up into four equal sections or if you had five sections of a board that was cut up into eight equal sections? *Figure 3* provides a visual reference for this question.

As you can see, it is hard to compare fractions that do not have common denominators, just as it is to mentally compare pieces of lumber cut into

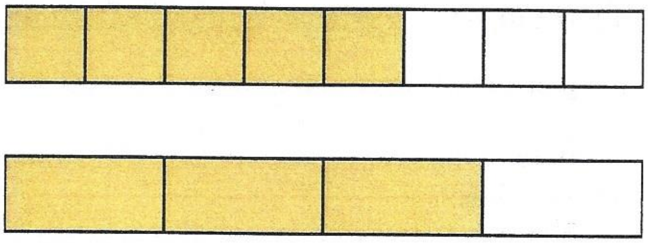

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Figure 3 Lumber divided into equal sections.

sections of different lengths. The visual reference in *Figure 3* makes it easy to see that $\frac{3}{4}$ of a board is more than $\frac{5}{8}$ of a board, but a visual reference like this is rarely available on the job. There is however, a mathematical way to make the task easier. If the fractions had the same denominator, then you would only need to look at the numerators of the two fractions to know which of them was larger.

$$\frac{3}{4} \text{ or } \frac{5}{8}$$

To easily compare them, you need to find a common denominator for the board sections. The common denominator is a number that both denominators can go into evenly.

Step 1 Multiply the two denominators together ($4 \times 8 = 32$). 32 is a common denominator between the two fractions, because each of the two denominators can be divided into it evenly. So a common denominator has been found that allows the two fractions to be compared more easily.

Step 2 Now finish converting the two fractions so that they will both have the same denominator of 32.

$$\frac{3}{4} \times \frac{8}{8} = \frac{24}{32}$$

$$\frac{5}{8} \times \frac{4}{4} = \frac{20}{32}$$

Now it is easy to compare the two fractions to see which is larger. You would have a longer section of lumber if you choose $\frac{3}{4}$ because you would have $\frac{24}{32}$ instead of $\frac{20}{32}$ of the lumber piece.

You have found a common denominator for this lumber problem. However, working with fractions like $\frac{24}{32}$ or $\frac{20}{32}$ can still be a little difficult. To make working with them easier, find the lowest common denominator possible between them, which means reducing the fractions to their lowest terms. Although 32 is a common denominator for the two fractions, it is not the lowest common denominator.

To find the lowest common denominator, follow these steps:

Step 1 Reduce each fraction to its lowest terms.

Step 2 Find the lowest common multiple of the denominators. Sometimes this is as simple as one denominator already being a multiple of the other. If this is the case, all you have to do is find the equivalent fraction for the term with the smaller denominator.

Step 3 If neither of the denominators is a multiple of the other, you can multiply the denominators together to get a common denominator.

Let's look at the 2×4 example again where $\frac{3}{4}$ and $\frac{5}{8}$ are already in their lowest terms. Upon looking at the denominators, you see that 8 is a multiple of 4. So the equivalent fraction for $\frac{3}{4}$ that has a denominator of 8 can be found this way:

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

You can now compare $\frac{6}{8}$ to $\frac{5}{8}$ and see that $\frac{6}{8}$ is the larger fraction. You could see this same basic result as $\frac{24}{32}$ and $\frac{20}{32}$ were compared. However, in this case, you are now working with the lowest common denominator.

Whether the lowest common denominator is determined or the denominators are simply multiplied together to find an acceptable common denominator will depend on the situation. In some applications, you may want all fractions involved to have a particular denominator. In the carpentry trade for example, working to within $\frac{1}{16}$" is common, so carpenters may tend to think of most fractional measurements in sixteenths.

Did You Know?

Nominal Measurements

A nominal 8" × 8" × 16" concrete block is actually $15\frac{5}{8}$" × $7\frac{5}{8}$" × $7\frac{5}{8}$" in dimension. It is referred to as an 8" × 8" × 16" block because of the addition of the $\frac{3}{8}$-inch mortar joint between the blocks. This measurement has been adopted as the trade standard for masonry joints. Adding the $\frac{3}{8}$ inch to the block's actual dimensions causes the block's measurements to come out in even inches, rather than as fractions. This holds true for other sizes and types of blocks.

2.1.4 Study Problems: Finding Equivalent Fractions

Find the equivalents of the following fractions:

1. $\frac{1}{4}$ equals how many sixteenths?
   a. $\frac{2}{16}$
   b. $\frac{4}{16}$
   c. $\frac{6}{16}$
   d. $\frac{8}{16}$
2. $\frac{2}{16}$ equals how many thirty-seconds?
   a. $\frac{1}{32}$
   b. $\frac{2}{32}$
   c. $\frac{4}{32}$
   d. $\frac{8}{32}$
3. $\frac{3}{4}$ equals how many eighths?
   a. $\frac{2}{8}$
   b. $\frac{4}{8}$
   c. $\frac{5}{8}$
   d. $\frac{6}{8}$
4. $\frac{3}{4}$ equals how many sixty-fourths?
   a. $\frac{48}{64}$
   b. $\frac{50}{64}$
   c. $\frac{52}{64}$
   d. $\frac{54}{64}$
5. $\frac{3}{16}$ equals how many thirty-seconds?
   a. $\frac{2}{32}$
   b. $\frac{4}{32}$
   c. $\frac{6}{32}$
   d. $\frac{8}{32}$

Find the lowest term for each of the following fractions without using a calculator.

6. $\frac{2}{16}$ = ________
7. $\frac{2}{8}$ = ________
8. $\frac{12}{32}$ = ________
9. $\frac{4}{8}$ = ________
10. $\frac{4}{64}$ = ________

Find the lowest common denominator for the following pairs of fractions.

11. $\frac{2}{6}$ and $\frac{3}{4}$.
    a. 6
    b. 10
    c. 12
    d. 16
12. $\frac{1}{4}$ and $\frac{3}{8}$.
    a. 4
    b. 8
    c. 12
    d. 18
13. $\frac{1}{8}$ and $\frac{1}{2}$.
    a. 3
    b. 5
    c. 7
    d. 8
14. $\frac{1}{4}$ and $\frac{3}{16}$.
    a. 8
    b. 16
    c. 18
    d. 20
15. $\frac{4}{32}$ and $\frac{5}{8}$.
    a. 64
    b. 32
    c. 16
    d. 8

2.2.0 Improper Fractions and Mixed Numbers

An **improper fraction** is defined as one in which the numerator is larger than the denominator. The fractions $\frac{43}{6}$, $\frac{19}{16}$, and $\frac{55}{8}$ are all improper fractions.

Improper fractions make better sense and are often more practical to work with when they are converted into a **mixed number**. A mixed number is one in which a whole number has been combined with a fraction. Based on the examples of improper fractions above, writing them as mixed numbers would result in $7\frac{1}{6}$, $3\frac{3}{16}$, and $6\frac{7}{8}$. Converting some improper fractions, such $\frac{42}{6}$, will result in a whole number alone, without an additional fraction. In this case, the fraction $\frac{42}{6}$ converts to the whole number 7.

As noted previously, fractions are really just another way of writing a division problem. To convert an improper fraction to a mixed number, the division problem is worked out. For this example, we will work with the improper fraction $\frac{67}{32}$. Here are the steps to convert an improper fraction to a mixed number:

Step 1 The numerator of the fraction (dividend) must be divided by the denominator (divisor). If the denominator cannot be divided into the numerator at least one full time, it is not an improper fraction. Set up a division problem if the numbers are difficult to handle mentally:

$$32\overline{)67}$$

Step 2 Complete the long division.

$$\begin{array}{r} 02 \text{ r3} \\ 32\overline{)67} \\ \underline{-64} \\ 3 \end{array}$$

Step 3 The quotient of the problem, 2, is the whole number of the mixed number. Any remainder becomes the numerator of the fraction, using the original denominator.

$2\frac{3}{32}$

2.2.1 Study Problems: Changing Improper Fractions to Mixed Numbers

Change these improper fractions to mixed numbers:

1. $35/8$ = ________.
2. $97/16$ = ________.
3. $13/4$ = ________.
4. $14/5$ = ________.
5. $48/16$ = ________.

2.3.0 Adding and Subtracting Fractions

To add fractions, it is necessary to find a common denominator and convert them to equivalent fractions. Once the denominators are the same, simple addition is used to add the numerators together. To add fractions, proceed as follows:

Step 1 Find the common denominator of the fractions you wish to add. A common denominator for ¾ and ⅝ is 32. The lowest common denominator is 8. The addition can be done using either one; it will not affect the accuracy of the result.

Step 2 Convert the fractions to equivalent fractions with the same denominator as shown.

$$\frac{3}{4} \times \frac{8}{8} = \frac{24}{32}$$

$$\frac{5}{8} \times \frac{4}{4} = \frac{20}{32}$$

Step 3 Add the numerators of the fractions. Place this sum over the denominator.

$$\frac{24}{32} + \frac{20}{32} = \frac{44}{32}$$

Step 4 Reduce the fraction to its lowest terms. In this example, both the numerator and denominator can be divided by 4. Doing so reduces the fraction to $11/8$. As you now know, this is an improper fraction. The improper fraction $11/8$ can be changed to the mixed number 1⅜.

As noted in *Step 1*, any common denominator can be used. However, when the lowest common denominator is used, the resulting sum of the fractions is already reduced to its lowest terms. Of course, the result may still be an improper fraction that must be changed to a mixed number.

Subtracting fractions is very much like adding fractions. You must find a common denominator before you can subtract. For example, assume you have ⅞ of a liter of paint left. If another worker asks for ¼ of it and you agree to share, how much will you have left?

Around the World

The Roots of Fractions

The word *fraction* originated from the Latin word *fractio* which means "to break." Other common words in the English language, such as *fracture*, were also derived from this Latin word. A number of different cultures, including the Egyptians, Babylonians, and the Indians all devised their own ways of considering and writing fractions over the years. It was the Arabs who added the line we now draw, sometimes on a slant and sometimes horizontally, between the numerator and the denominator.

Step 1 Find a common denominator. In this case, 8 is a common denominator and is also the lowest common denominator.

$$\frac{7}{8} \quad \frac{1}{4}$$

Step 2 Since one fraction is already in eighths, only the ¼ needs to converted to eighths. Multiply each term (numerator and denominator) by 2 to get a fraction with the denominator of 8.

$$\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$$

Step 3 With both fractions using the same denominator, the numerators of the two fractions can be subtracted. This results in a difference of ⅝.

$$\frac{7}{8} - \frac{2}{8} = \frac{5}{8}$$

Sometimes a fraction must be subtracted from a whole number. This requires that the whole number be changed to a fraction as well. Once it is a fraction, find a common denominator and subtract as before. In this example, 2⁄7 will be subtracted from 5.

Changing any whole number into a fraction is very simple. By simply adding a 1 as a denominator, a whole number becomes a fraction. In this case, the whole number 5 becomes the fraction 5⁄1. The lowest common denominator between 5⁄1 and 2⁄7 is 7. Converting 5⁄1 into sevenths results in the fraction 35⁄7. Now the two fractions can be subtracted easily, resulting in a difference of 33⁄7. This improper fraction can be changed to the mixed number of 4 5⁄7.

2.3.1 Study Problems: Adding and Subtracting Fractions

Find the answers to the following addition problems. Remember to reduce the sum to the lowest terms and change any improper fractions to mixed numbers.

1. ⅛ + 4⁄16 = ________
2. 4⁄8 + 6⁄16 = ________
3. 2⁄4 + ¾ = ________
4. ¾ + 2⁄8 = ________
5. 14⁄16 + ⅜ = ________

Find the answers to the following subtraction problems. Remember to reduce the differences to the lowest terms.

6. ⅜ − 5⁄16 = ________
7. 11⁄16 − ⅝ = ________
8. ¾ − 2⁄6 = ________
9. 11⁄12 − 4⁄8 = ________
10. 11⁄16 − ½ = ________

Find the answers to the following subtraction problems involving fractions and whole numbers. Reduce the fractions to their lowest terms and change any improper fractions to mixed numbers.

11. 8 − ¾ = ________
12. 12 − ⅝ = ________
13. Two punches are made from steel bar stock 9 7⁄16 inches long. If one punch is 4 1⁄64 inches long and the other is 4 3⁄32 inches long, how many inches of stock are wasted?
    a. 1 1⁄16 inches
    b. 1 5⁄16 inches
    c. 1 15⁄64 inches
    d. 1 21⁄64 inches
14. If you saw 12 1⁄16 inches off a board that is 20¾ inches long, the length of the remaining board will be _____.
    a. 8¼ inches
    b. 8 11⁄16 inches
    c. 11¼ inches
    d. 17⅜ inches
15. A rough opening for a window measures 36⅜ inches. The window to be placed in the rough opening measures 35 15⁄16 inches. The total clearance that should exist between the window and the rough opening will be _____.
    a. 7⁄16 inch
    b. 1 inch
    c. 1 7⁄16 inches
    d. 1¾ inches

2.4.0 Multiplying and Dividing Fractions

Multiplying and dividing fractions is very different from adding and subtracting fractions. You do not have to find a common denominator when you multiply or divide fractions.

In a word problem, the words used let you know if you need to multiply. If a problem asks

"What is $\frac{2}{8}$ of 9?" then think of the problem this way: $\frac{2}{8} \times \frac{9}{1}$. Remember that any number (except 0) over 1 equals itself. Therefore, $\frac{9}{1}$ is simply a fractional way to write the whole number 9.

Using $\frac{4}{8} \times \frac{5}{6}$ as an example, follow these steps:

Step 1 Multiply the numerators together to get a new numerator. Multiply the denominators together to get a new denominator.

$$\frac{4}{8} \times \frac{5}{6} = \frac{20}{48}$$

Step 2 Multiplication often results in fractions that are not in their lowest terms. Reduce the product if possible. In this example, $\frac{20}{48}$ can be reduced to $\frac{5}{12}$, since both the numerator and denominator are divisible by 4.

Although you can multiply fractions without first reducing them to their lowest terms, you can reduce them before you multiply. This will sometimes make the multiplication easier, since you will be working with smaller numbers. It will also make it easier to reduce the product to the lowest terms. What may seem like an extra step can save you time in the long run. Regardless of which way is chosen, the same product will result.

Dividing fractions is very much like multiplying fractions, with one difference. You must **invert**, or flip, the fraction you are dividing by. Using $\frac{1}{2} \div \frac{3}{4}$ as an example, follow these steps:

Step 1 Invert the fraction that is the divisor. In our example, $\frac{3}{4}$ is the divisor. Therefore:

$$\frac{3}{4} \text{ becomes } \frac{4}{3}$$

Step 2 Now change the division sign (÷) to a multiplication sign (×).

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3}$$

Step 3 Multiply the fraction as described earlier.

$$\frac{1}{2} \times \frac{4}{3} = \frac{4}{6}$$

Step 4 Reduce to the lowest terms when possible.

$$\frac{4}{6} \text{ reduce to } \frac{2}{3}$$

Thus, $\frac{3}{4}$ will go into $\frac{1}{2}$ two-thirds of a time. A division problem can be checked with multiplication to ensure its accuracy. Multiplying $\frac{3}{4}$ times $\frac{2}{3}$ results in a product of $\frac{1}{2}$—the original dividend of the problem.

If you are working with a mixed number, $2\frac{1}{3}$ for example, it must be converted into a fraction before it is inverted. Do this by multiplying the denominator of the fraction (3) by the whole number (2). Then add the numerator of the fraction and place the result over the denominator. That portion looks like this:

$$[(3 \times 2) + 1]$$

The result of the math above, 7, becomes the numerator of the fraction. The original denominator remains in place. The complete process of changing the mixed number $2\frac{1}{3}$ to a fraction looks like this:

$$2^1/_3 = \frac{(3 \times 2) + 1}{3} = \frac{7}{3}$$

When dividing a fraction by a whole number, first place the whole number over a 1 to convert it to a fraction. Remember that $\frac{4}{1}$ is the same as 4. Then invert the fraction. For example:

$$^1/_2 \div 4$$
$$^1/_2 \div {}^4/_1$$
$$^1/_2 \times {}^1/_4 = {}^1/_8$$

2.4.1 Study Problems: Multiplying and Dividing Fractions

Find the answers to the following multiplication problems without using a calculator. Reduce the products to their lowest terms and change improper fractions to mixed numbers.

1. $\frac{4}{16} \times \frac{5}{8}$ = ________
2. $\frac{3}{4} \times \frac{7}{8}$ = ________
3. $\frac{2}{8} \times 15$ = ________
4. $\frac{3}{7} \times 49$ = ________
5. $\frac{8}{16} \times \frac{32}{64}$ = ________

Find the answers to the following division problems without using a calculator. Reduce the quotients to their lowest terms and change improper fractions to mixed numbers.

6. $\frac{3}{8} \div 3$ = ________
7. $\frac{5}{8} \div \frac{1}{2}$ = ________
8. $\frac{3}{4} \div \frac{3}{8}$ = ________

9. On construction drawings, smaller dimensions are often used to represent larger ones. This allows the large object or structure to fit on the paper. On such a drawing, if ¼-inch represents a distance of 1 foot, then a line on the drawing measuring 8½ inches would represent how many feet?
   a. 34
   b. 36
   c. 38
   d. 40

10. How many ⅞-inch long strips can be cut from a single 7-inch long strip of material?
   a. 5
   b. 6
   c. 7
   d. 8

Additional Resources

Applied Construction Math: A Novel Approach, NCCER. 2006. Upper Saddle River, NJ: Prentice Hall.

Mathematics for Carpentry and the Construction Trades, Alfred P. Webster; Kathryn B. Judy. 2001. Upper Saddle River, NJ: Prentice Hall.

Mathematics for the Trades: A Guided Approach, Robert A. Carman; Hal Saunders. 2014. Pearson Learning.

2.0.0 Section Review

1. Which of the following fraction pairs are equivalent fractions?
   a. ½ and 17/32
   b. ½ and 32/64
   c. ½ and ¾
   d. ½ and 63/128

2. The improper fraction 76/64 converts to the mixed number 1 9/32.
   a. True
   b. False

3. 11/16 + ⅜ = _____.
   a. 9/16
   b. ⅞
   c. 1 1/16
   d. 1⅛

4. 1¼ − 7/16 = _____.
   a. 5/16
   b. ⅝
   c. ¾
   d. 13/16

5. ⅛ × 1/10 = _____.
   a. 1/80
   b. 1/40
   c. ⅘
   d. 1¼

6. 7/16 ÷ ⅞ = _____.
   a. 49/128
   b. ⅜
   c. ½
   d. 2

Section Three

3.0.0 The Decimal System

Objective

Describe the decimal system and explain how to work with decimals.

a. Describe decimals and their place values.
b. Demonstrate the ability to add, subtract, multiply, and divide decimals.
c. Demonstrate the ability to convert between decimals, fractions, and percentages.

Decimals are based entirely on the number 10. Through learning the place values of digits in a given number earlier in this module, you have learned half of the decimal system already. Whole numbers, such as 26,493, do not typically have a decimal point written beside them. However, that does not mean a decimal point is not actually there. When a decimal point is not visible, one is always assumed to be to the right of the last number. For the number 26,493, a decimal point is assumed to be on the right side of the number 3.

3.1.0 Decimals

By using digits placed to the right of a decimal point, a part of a number can be written. Proper fractions represent part of the number 1. When using decimals instead of fractions, all numbers written to the right of a decimal point also represent part of the number 1. In fact, decimals are often referred to as decimal fractions, since they represent part of a whole number.

Figure 4 shows the place values of digits placed on the left and on the right of the decimal point.

Of course, place values extend far beyond what is shown, in both directions. For applications requiring extreme precision, extending a number five places to the right of the decimal point usually provides the required precision. For more common applications, the precision offered by extending a number one or two places to the right of the decimal point is often sufficient. This depends entirely on the application. In some cases, even a whole number is sufficient.

To read a decimal, say the number as it is written and then the name of the place value for the right-most digit. For example, 0.56 is spoken as fifty-six hundredths. The right-most digit, the 6, is located in the hundredths place.

Note that the number written for this example (0.56) begins with a zero on the left of the decimal point. Although it is not required to properly read the decimal value, it is common to fill the first position to the left of the decimal with a zero for clarity when the decimal represents a value less than one.

Mixed numbers also appear in decimals. The number 15.7 is an example. It is read as fifteen and seven-tenths. Notice the use of the word "and" to separate the whole number from the decimal.

3.1.1 Rounding Decimals

Sometimes an answer is a bit more precise than required. As an example, if tubing costs €3.76 (the € symbol represents the euro, currency of the European Union) per meter and €800 has been budgeted for the tubing, how much tubing can be purchased?

The precise mathematical answer is 212.7659574 meters. However, it is quite unlikely that 0.7659574 meters of tubing can be purchased. A value to the nearest tenth is sufficient for this application. For this exercise, 212.7659574 will be rounded to the nearest tenth. This represents only one position to the right of the decimal point.

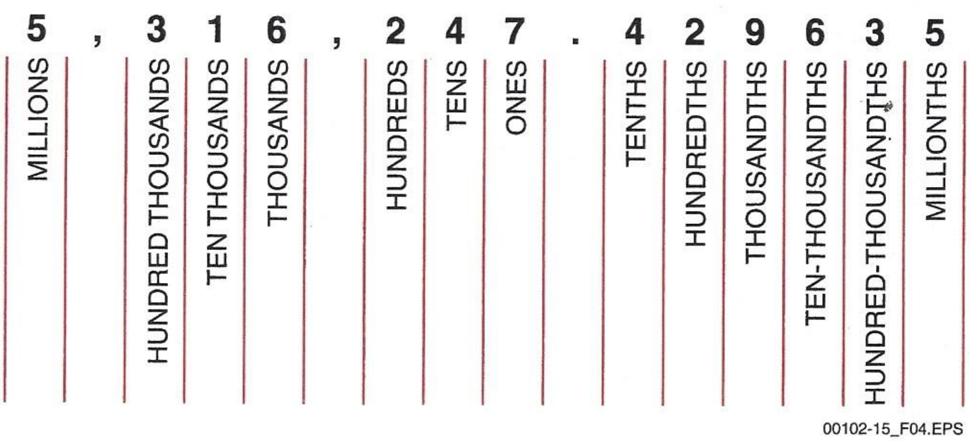


Figure 4 Place values on both sides of the decimal point.

Step 1 Underline the place to which you are rounding.

212.7659574

Step 2 Look at the digit one place to its right.

212.7659574

Step 3 If the digit to the right is 5 or more, you will round up by adding 1 to the underlined digit. If the digit is 4 or less, leave the underlined digit the same. In this example, the digit to the right is 6, which is more than 5, so you round up by adding 1 to the underlined digit. Then drop all the remaining digits to the right of it.

212.8

3.1.2 Comparing Decimals with Decimals

It is relatively simple to tell if one decimal value is larger or smaller than another. It is really no different than determining if one whole number is larger than another. You should be able to tell which of these numbers is larger on sight:

4,214 or 4,217

Obviously 4,217 is the larger of the two numbers. A decimal point can be added at any position to make this number a decimal.

0.4214 or 0.4217

The result is the same. The number with the digits 4217 is larger than the other.

3.1.3 Study Problems: Working With Decimals

For the following problems, identify the words that represent the proper way to speak the decimal value shown.

1. 0.4 = _____.
   a. four
   b. four-tenths
   c. four-hundredths
   d. four-thousandths
2. 0.05 = _____.
   a. five
   b. five-tenths
   c. five-hundredths
   d. five-thousandths
3. 2.5 = _____.
   a. two and five-tenths
   b. two and five-hundredths
   c. two and five-thousandths
   d. twenty-five-hundredths

For the following problems, identify the written numerical value of the spoken number.

4. Eighteen-hundredths = _____.
   a. 0.0018
   b. 0.018
   c. 0.18
   d. 1.8
5. Five and eight-tenths = _____
   a. 5.0
   b. 5.008
   c. 5.08
   d. 5.8

For the following problems, select the answer that places the decimals in order from smallest to largest.

6. 0.400, 0.004, 0.044, and 0.404
   a. 0.400, 0.004, 0.044, 0.404
   b. 0.004, 0.044, 0.404, 0.400
   c. 0.004, 0.044, 0.400, 0.404
   d. 0.404, 0.044, 0.400, 0.004
7. 0.567, 0.059, 0.56, and 0.508
   a. 0.508, 0.56, 0.567, 0.059
   b. 0.059, 0.56, 0.508, 0.567
   c. 0.567, 0.059, 0.56, 0.508
   d. 0.059, 0.508, 0.56, 0.567
8. 0.320, 0.032, 0.302, and 0.003
   a. 0.003, 0.032, 0.302, 0.320
   b. 0.320, 0.302, 0.032, 0.003
   c. 0.302, 0.320, 0.003, 0.032
   d. 0.003, 0.032, 0.320, 0.302
9. 0.867, 0.086, 0.008, and 0.870
   a. 0.870, 0.867, 0.086, 0.008
   b. 0.008, 0.086, 0.867, 0.870
   c. 0.086, 0.008, 0.867, 0.870
   d. 0.008, 0.870, 0.867, 0.086
10. 0.626, 0.630, 0.616, and 0.641
    a. 0.616, 0.641, 0.630, 0.626
    b. 0.616, 0.626, 0.630, 0.641
    c. 0.641, 0.616, 0.626, 0.630
    d. 0.630, 0.616, 0.626, 0.641

3.2.0 Adding, Subtracting, Multiplying, and Dividing Decimals

In many ways, mathematical operations with decimals are not significantly different than working with whole numbers. Placing the decimal point in the proper location is however, essential to finding the correct answer.

3.2.1 Adding and Subtracting Decimals

The most important rule to remember when adding and subtracting decimals is to keep the decimal points aligned as the problem is written.

In this example, 4.76 and 0.834 will be added together. Note that these two numbers have a different number of digits to the right of the decimal. Line up the problem as shown here, adding a 0 if needed to help keep the numbers lined up.

$$\begin{array}{r} 4.760 \\ +\ 0.834 \\ \hline 5.594 \end{array}$$

Note that the decimal points of the two numbers being added are aligned vertically. The decimal point of the sum is also aligned with them. The same thing is true for subtraction of decimals. To subtract 2.724 from 5.6, line up the decimal points as shown.

$$\begin{array}{r} 5.600 \\ -\ 2.724 \\ \hline 2.876 \end{array}$$

Did You Know?

Digits and Decimals

When a number is written, a symbol representing that number is marked or printed. This symbol is often referred to as a digit. The word *digit* is derived from the Latin word for finger.

Early people (and a lot of us inhabiting the planet today) naturally used their fingers as a means of counting—a base-ten counting system, since we have ten fingers. The decimal system naturally evolved from this. The word *decimal* is also derived from a Latin word. In Latin, decimal means "ten."

Notice that two zeros were added to the end of the first number to make it easier to see where you need to borrow. This is not required, but it does help to ensure that the problem is properly aligned.

3.2.2 Multiplying Decimals

A series of partitions must be set up in an office. One partition is measured to determine its width, which is 4.5 feet. There are seven panels of the same width. How many feet of partition can be erected if the panels are standing side-by-side?

Step 1 Set up the problem just like the multiplication of whole numbers.

$$\begin{array}{r} 4.5 \\ \times\ 7 \\ \hline \end{array}$$

Step 2 Proceed to multiply.

$$\begin{array}{r} 4.5 \\ \times\ 7 \\ \hline 315 \end{array}$$

Step 3 Once you have the answer, count the number of digits to the right of the decimal point in both of the numbers being multiplied. In this example, there is only one with a number to the right of the decimal point (4.5), and there is only one digit to the right of it.

Step 4 In the answer, count over the same number of digits, from right to left, and place the decimal point there.

$$\begin{array}{r} 4.5 \\ \times\ 7 \\ \hline 31.5 \end{array}$$

You may have to add one or more zeros if there are more digits to the right of the decimal points than there are in the answer, as shown in the following example.

$$\begin{array}{r} 0.507 \\ \times\ 0.022 \\ \hline 1014 \\ 10140 \\ 000000 \\ +\ 0000000 \\ \hline 11154 \end{array} = 0.011154$$

Add the digits to the right of the decimal point in the two numbers. There are six total. Count six digits from right to left in the product. In this case, you'll need to add a zero.

3.2.3 Dividing with Decimals

There are three types of division problems involving decimals:

- Those that have a decimal point in the number being divided (the dividend):

$$22\,\overline{)\,44.5}$$

- Those that have a decimal point in the number you are dividing by (the divisor):

$$0.22\,\overline{)\,4{,}450}$$

- Those that have decimal points in both numbers (the dividend and the divisor):

$$0.22\,\overline{)\,44.5}$$

The examples above will be used to demonstrate each type of decimal division problem.

For the first type of problem where a decimal point is shown only in the dividend, follow these steps:

Step 1 Place a decimal point directly above the decimal point in the dividend.

$$22\,\overline{)\,44.5}$$

Step 2 Divide as usual.

$$\begin{array}{r} 02.0 \\ 22\,\overline{)\,44.5} \\ \underline{-\,44} \\ 00.5\text{r} \end{array}$$

The answer is 2, with a remainder of 0.5. Note again the alignment of the decimal point in the quotient, directly above the decimal point in the dividend. This is critical for this type of problem.

For the second type of problem, where only a decimal point is found in the divisor, follow these steps:

Step 1 Move the decimal point in the divisor to the right until you have a whole number.

0.22 becomes 22

Step 2 Next, move the decimal point in the dividend the same number of places (two) to the right. Zeros will have to be added, since the dividend is a whole number. Now place the decimal point in the quotient directly above the one in the dividend. Then divide as usual.

$$\begin{array}{r} 20227.2 \\ 22\overline{)4450.00.0} \\ \underline{-44} \\ 0050 \\ \underline{-0044} \\ 00060 \\ \underline{-00044} \\ 000160 \\ \underline{-000154} \\ 0000060 \\ \underline{-0000044} \\ 0000016\text{r} \end{array}$$

Remember that division like this can always be checked by multiplying the quotient times the divisor, and then adding the remainder. The answer should always result in the dividend.

For the third type of problem, where there is a decimal point in both the dividend and the divisor, follow these steps:

Did You Know?

Scientific Notation

Scientific notation (sometimes called exponential notation) is a system that allows you to conveniently write very large or very small decimal-based numbers using an exponent. The exponent represents the number of times you multiply the multiplier by the multiplicand. Scientific notation is commonly used by scientists, mathematicians, and engineers. You may already be familiar with scientific notation if you used ft^2 (feet squared) in a measurement. The following are some examples of scientific notation:

Decimal Notation	Scientific Notation
1	1×10^0
50	5×10^1
7,530,000,000	7.53×10^9
-0.0000000082	-8.2×10^{-9}

Step 1 Move the decimal point in the divisor to the right until you have a whole number.

0.22 becomes 22

Step 2 Move the decimal point in the dividend the same number of places to the right. 0.22 and 44.5 now become 22 and 4,450, respectively. By moving the decimal point in both the divisor and the dividend, the quotient will not be affected.

$$22\overline{)44.50.}$$

Step 3 Then divide as usual.

$$\begin{array}{r} 202 \\ 22\,\overline{)\,4450} \\ \underline{-\,44} \\ 005 \\ \underline{-\,000} \\ 0050 \\ \underline{-\,0044} \\ 0006r \end{array}$$

The answer is 202 with a remainder of 6.

3.2.4 Using the Calculator to Add, Subtract, Multiply, and Divide Decimals

Performing operations on the calculator using decimals is very much like performing the operations on whole numbers. Follow these steps using the problem 45.6 + 5.7 as an example.

Step 1 Turn the calculator on.

Step 2 Press 4, 5, . (decimal point), and 6. The number 45.6 appears in the display.

NOTE

For this step, press whichever operation key the problem calls for: + to add, − to subtract, × to multiply, ÷ to divide.

Step 3 Press the + key. The 45.6 is still displayed.

Step 4 Press 5, . (decimal point), and 7. The number 5.7 is displayed.

Step 5 Press the = key. After you press the = key, whether you are adding, subtracting, multiplying, or dividing, the answer will appear on your display.

$$45.6 + 5.7 = 51.3$$
$$45.6 - 5.7 = 39.9$$
$$45.6 \times 5.7 = 259.92$$
$$45.6 \div 5.7 = 8$$

Step 6 Press the On or C key to fully clear the calculator. The CE key clears only the most recent entry. A zero (0) appears in the display.

Did You Know?

Other Counting Systems

Ancient Babylonians developed one of the first place-value systems—a base-sixty system. It was based on the number 60, and numbers were grouped by sixties. At the beginning, there was no symbol for zero in the Babylonian system. This made it difficult to perform calculations. It was not always possible to determine if a number represented 24, 204, or 240. Although this system is no longer used today, we still use a base-sixty system for measuring time: 60 seconds in a minute and 60 minutes in an hour.

A base-twelve system requires 12 digits. This system is called a duodecimal system, from the Latin word *duodecim*, meaning "twelve." Since the decimal system has only ten digits, two new digits must be added to the base-twelve system. In a duodecimal system, each place value is 12 times greater than the place to the right. Although this is not a common system, a base-twelve system is used to count objects by the dozen (12) or by the gross (144 = 12 × 12).

A hexadecimal system groups numbers by sixteens. The word *hexadecimal* comes from the Greek word for "six" and the Latin word for "ten." Just as the duodecimal system required two new place numerals, the hexadecimal system requires six additional digits. In a hexadecimal system, each place value is 16 times greater than the place to the right. Computers often use the hexadecimal system to store information. Common configurations of random access memory (RAM) come in multiples of 16. 1,024 kilobytes (kB) equals 1 megabyte (MB).

3.2.5 Study Problems: Decimals

Find the answers to these addition and subtraction problems without using a calculator. Be sure to maintain vertical alignment of the decimal points.

1. 2.50 + 4.20 + 5.00 = ________
2. 1.82 + 3.41 + 5.25 = ________
3. 6.43 + 86.4 = ________
4. The combined thickness of a piece of sheet metal 0.078 centimeters (cm) thick and a piece of band iron 0.25 cm thick is ________.
   a. 0.308 cm
   b. 0.328 cm
   c. 3.08 cm
   d. 32.8 cm
5. Yesterday, a lumber yard contained 6.7 tons of wood. Since then, 2.3 tons were removed. How many tons of wood remain?
   a. 3.4 tons
   b. 4.4 tons
   c. 5.4 tons
   d. 6.4 tons

Use the following information to answer Questions 6 and 7:

A part is being machined. The starting thickness of the part is 6.18 inches. Three cuts are taken. Each cut is three-tenths of an inch.

6. How many inches of material have been removed at this point?
   a. 0.6 inches
   b. 0.8 inches
   c. 0.9 inches
   d. 1.09 inches
7. The remaining thickness of the part is _____.
   a. 5.28 inches
   b. 6.08 inches
   c. 6.10 inches
   d. 6.15 inches
8. Ceramic tile weighs 4.75 pounds per square foot. Therefore, 128 square feet of ceramic tile weighs _____.
   a. 598 pounds
   b. 608 pounds
   c. 908 pounds
   d. 1108 pounds

Find the answers to the following division problems without using a calculator. Answers should be rounded to the nearest hundredth.

9. 45.36 ÷ 18 = ________
10. 4.536 ÷ 18 = ________
11. 0.4536 ÷ 18 = ________

Round the quotients of these problems to the nearest hundredth.

12. $25\overline{)10.20}$
13. $6\overline{)31.2}$

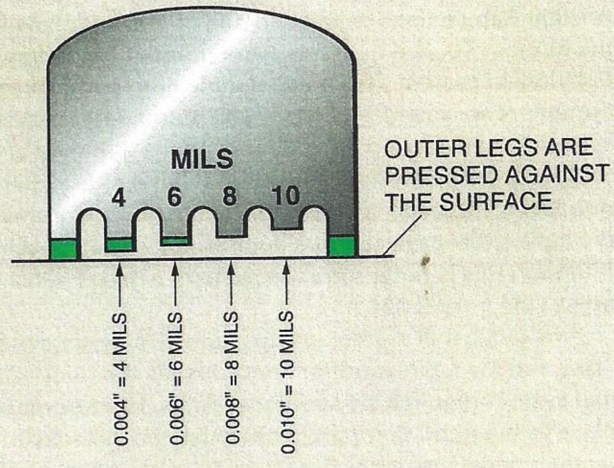
Perform the following division problems on a separate piece of paper without using a calculator. Note that a decimal point is only present in the divisors of these problems. Answers should be rounded to the nearest hundredth.

14. 282 ÷ 14.1 = ________
15. 694 ÷ 3.2 = ________
16. 99 ÷ 0.45 = ________

Measuring the Thickness of a Coating

Coating thickness is important on many parts and surfaces because too little or too much can cause problems. A coating such as paint needs a minimum thickness to prevent corrosion, withstand abrasion, and look good. A coating that is too thick may crack, flake, blister, or not cure properly.

The thickness of the coating to be applied to an object or surface is often specified by the buyer or client. To ensure that the specifications are met, periodic checks of the wet-film thickness can be made using a wet-film thickness gauge. Typically these gauges use measurements in mils or microns. As shown here on a typical gauge, a mil is equal to one one-thousandth of an inch (0.001), which can also be written as 10^{-3}. A mil is also equal to 0.0254 millimeters.



Thickness is determined by the longest tooth with paint and the shortest tooth without paint. The coating thickness here is 6-8 mils.

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17. $2.5\overline{)102}$

18. $0.6\overline{)312}$

Find the answers to the following division problems without using a calculator. Note that a decimal point is present in both the divisor and the dividend of these problems. Answers should be rounded to the nearest hundredth.

19. $20.82 \div 4.24 =$ ________

20. $38.9 \div 3.7 =$ ________

21. $9.9 \div 0.45 =$ ________

22. $0.25\overline{)10.20}$

23. $0.6\overline{)31.2}$

Use your calculator to find the answers to the following problems, rounding your answers to the nearest hundredth.

24. $\begin{array}{r} 45.89 \\ \underline{+\ 7.85} \end{array}$

25. $\begin{array}{r} 7.6 \\ \underline{\times\ 0.12} \end{array}$

26. $\begin{array}{r} 685.79 \\ \underline{-\ 56.266} \end{array}$

27. $6.45 \div 3.25 =$

28. $\begin{array}{r} 34.76 \\ \underline{+\ 3.64} \end{array}$

Solve these problems to practice rounding decimals. Round your answers to the nearest tenth.

29. You need to cut a 90.5-inch pipe into as many 3.75-inch pieces as possible. How many complete 3.75-inch pieces will you be able to cut?
   a. 14
   b. 24
   c. 34
   d. 44

30. If a car traveled 1,001 kilometers on 151.8 liters of gas, how many kilometers per liter would it be achieving, to the nearest tenth of a kilometer?
   a. 0.1
   b. 0.2
   c. 6.5
   d. 6.6

31. If wire costs $4.30 per pound and you pay a total of $120.95, how many pounds of wire were purchased, to the nearest hundredth of a pound?
   a. 0.28
   b. 2.8
   c. 28.1
   d. 28.13

32. One size of vent pipe is on sale at XYZ Supply Company this week for $0.37 per linear foot. How many feet of vent pipe can be purchased with $115.38, to the nearest tenth of a foot?
   a. 308.1
   b. 310.8
   c. 311.8
   d. 311.9

33. Vent pipe at XYZ Supply costs $0.48 per linear foot when it is not on sale. If you spend $115.38, how many feet can be purchased, to the nearest tenth of a foot?
   a. 240.38
   b. 240.4
   c. 241
   d. 241.4

3.3.0 Converting Decimals, Fractions, and Percentages

Sometimes numbers need to be converted from one form to another to make them easier to work with. Some numbers relevant to the job at hand may appear as decimals, some as percentages, and others as fractions. Decimals, percentages, and fractions are all just different ways of expressing the same thing. The decimal 0.25, the percent 25%, and the fraction ¼ all represent the same numerical value or quantity. In order to work with different forms of numbers, they must be converted from one form into another.

3.3.1 Converting Decimals to Percentages and Percentages to Decimals

To understand percentages, think of a whole number divided into 100 parts. Thinking about a $1.00 bill is a perfect way to understand percentages, since it is broken into 100 equal parts (100 pennies). Each penny represents 1% of a dollar.

Any part of a whole can be expressed as a percentage. *Figure 5* provides an example more appropriate for the trades. The tank has a capacity of 100 gallons. It is now filled with 50 gallons. To what percentage is the tank filled?

The correct answer is 50%. The percentage reflects a quantity based on the there being 100 parts to something. How many gallons out of 100 does the tank contain? It contains 50 out of 100, or 50 percent. Percentages are an easy way to express parts of a whole.

Decimals and fractions also express parts of a whole. The tank in *Figure 5* is 50 percent full. If you expressed this as a fraction, you would say it is ½ full. You could also express this as a decimal and say it is 0.50 full. In fact, a percentage can be expressed as a decimal as well. If a half-gallon of liquid is added to the tank in *Figure 5*, it would be 50.5% full.

Sometimes decimals need to be expressed as percentages, or percentages as decimals. Suppose a gallon of cleaning solution needs to be prepared.

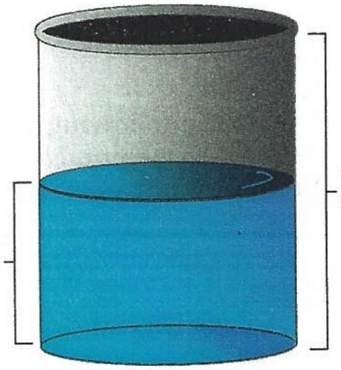

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Figure 5 100-gallon tank.

The mixture should be comprised of 10% to 15% of the cleaning agent. The rest should be water. There is 0.12 gallon of cleaning agent concentrate available. Will this be enough to prepare a gallon of the solution? To answer the question, the decimal value of the concentrate (0.12) needs to be converted to a percentage. Follow these steps to make the conversion:

Decimals at Work

The use of decimal measurements is very important to a number of trades, including millwrights. Millwrights are often tasked with aligning pump and drive motor shafts. To eliminate vibration and prevent damage to the drive coupling and pump seal, many shafts must be aligned with extreme accuracy. This pump and motor are being aligned using laser technology. The level of precision is specified by the pump manufacturer, often to within 0.002 to 0.003 of an inch. Shims of various thicknesses are purchased or fabricated in various thicknesses and placed under the feet of the pump and/or motor to achieve alignment.

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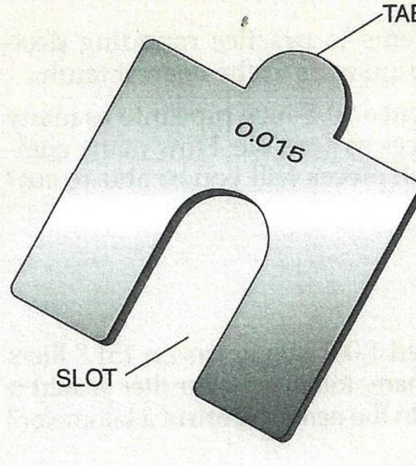

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Step 1 Multiply the decimal by 100. (Tip: When multiplying by 100, simply move the decimal point of the number two places to the right.)

$$0.12 \times 100 = 12$$

Step 2 Add a % sign.

12%

Recall that the mixture should be from 10% to 15% cleaning agent. Since there is 12% of a gallon available, there is enough cleaning agent to make a gallon of solution.

You may also need to convert percentages to decimals. Use the following as an example. Suppose a mixture should contain 22% of a certain chemical by weight. One pound of the mixture is to be made. The ingredients are weighed on a digital scale, which cannot be read in percentages of a pound but can be read in decimal portions of a pound. To determine how much chemical is needed in decimal form, convert the percentage (22%) to a decimal by following these steps:

Step 1 Drop the % sign.

22

Step 2 Divide the number by 100. (Tip: When dividing by 100, simply move the decimal point two places to the left.)

$$22 \div 100 = 0.22$$

Therefore, add 0.22 pound of the chemical to 0.78 pound of the other ingredient(s) to make one pound at a 22% mixture.

3.3.2 Converting Fractions to Decimals

You will often need to change a fraction to a decimal. Remember that a fraction is already a division problem that has been written differently. To change a fraction to a decimal value, simply complete the division problem. The following steps show how it is done for the fraction ¾, as in ¾ of a liter:

Step 1 Divide the numerator of the fraction by the denominator.

$$4\overline{)3.0}$$

In this example, place a decimal point and the zero after the number 3. Since 4 will not divide into 3, more digits will be needed to work the problem.

Step 2 Place the decimal point for the quotient directly above its location in the dividend.

$$\begin{array}{r} . \\ 4\overline{)3.0} \end{array}$$

Step 3 Now divide as you normally would.

$$\begin{array}{r} .75 \\ 4\overline{)3.00} \\ \underline{-\ 2.8} \\ 0.20 \\ \underline{-\ 0.20} \\ 0.00 \end{array}$$

The decimal equivalent of ¾ is 0.75. As demonstrated earlier, this value can be turned into a percentage by moving the decimal point two positions to the right. So ¾ of a liter is also 0.75 liters or 75% of a liter.

3.3.3 Converting Decimals to Fractions

Converting a decimal to a fraction is relatively simple. Remember that both decimals and fractions are different ways to express the same thing. When a decimal value is spoken, it is spoken like a fraction. When it is written the same way it is spoken, it automatically translates as a fraction.

Drill bits come in fractional sizes, decimal sizes, and even sets identified by numbers and letters alone. Assume a hole that is to be drilled is specified as 0.125 inch in size. However, all you have are fractional drill sizes. It is possible that a fractional drill size is actually the same size. To find out, follow these steps to convert the decimal 0.125 to a fraction:

Step 1 Say the decimal in words.

0.125 is one hundred twenty-five thousandths

Step 2 Write the decimal as a fraction, just like it was spoken.

0.125 written as a fraction is $^{125}/_{1000}$

Step 3 Reduce the fraction to its lowest terms.

$$\frac{125}{1000} = \frac{125}{1000} \div \frac{125}{125} = \frac{1}{8}$$

0.125 converted to a fraction is ⅛. If there is a ⅛" drill in the box, the specification has been met and the hole can be drilled.

3.3.4 Converting Inches to Decimal Equivalents in Feet

Unlike the United States, the vast majority of the world works with the metric system of measurement. The metric system is perfectly matched to the decimal system, since both are based on units of 10. However, for those working with the Imperial or inch-pound system of measurement, measurements may need to be converted to decimals. For example, what decimal part of a foot does 3 inches represent?

First, express the inches as a fraction that has 12 as the denominator. This is accurate because an inch represents $\frac{1}{12}$th of a foot. The fraction for 3 inches is written as $\frac{3}{12}$.

In this example, the fraction $\frac{3}{12}$ can be reduced to lower terms—$\frac{1}{4}$. Convert the fraction $\frac{1}{4}$ to a decimal by dividing the 4 into 1.00:

$$\begin{array}{r} .25 \\ 4\overline{)\,1.00} \\ -\,0.8 \\ \hline 0.20 \\ -\,0.20 \\ \hline 0.00 \end{array}$$

3 inches is equal to 0.25 foot.

3.3.5 Study Problems: Converting Different Values

Convert these decimals to percentages.

1. 0.62 = ________
2. 0.475 = ________
3. 0.7 = ________

Convert these percentages to decimals.

4. 72% = ________
5. 12.5% = ________

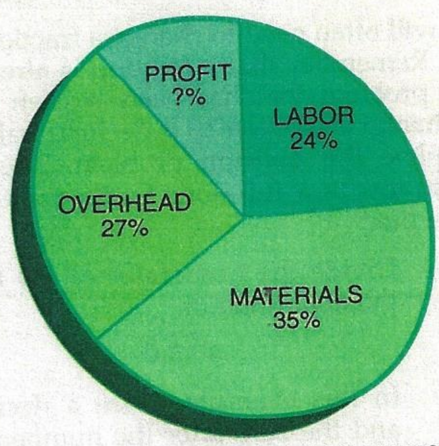
Convert the following fractions to their decimal equivalents without using a calculator.

6. $\frac{1}{4}$ = ________
7. $\frac{3}{4}$ = ________
8. $\frac{1}{8}$ = ________
9. $\frac{5}{16}$ = ________
10. $\frac{20}{64}$ = ________

Practical Math Application

When contractors calculate the expenses for building a house, they must pay close attention to the percentages they allow for various factors. Of course, the profit realized by the company is an important consideration. For example, if a contractor is building a house that will have a selling price of $175,000, what will the profit be with the following percentage breakdown?

- Profit =?%
- Overhead = 27%
- Materials = 35%
- Labor = 24%



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Solution

Since $175,000 represents 100% of the revenue, the total cost and profit together must equal 100%. Adding together the known percentages above reveals that the total cost of the job was 86% of the revenue. That means the profit must be 14% of the revenue. The profit, in dollars, can now be found this way:

$$\text{Profit} = \$175{,}000 \times 14\% = \$175{,}000 \times 0.14 = \$24{,}500$$

You can also use your calculator to work with percentages. For example, to solve the profit component of this problem, key it into your calculator as follows:

Step 1 Punch 175000 into the calculator.
Step 2 Press the multiplication button.
Step 3 Punch 14 into the calculator.
Step 4 Press the percent (%) button.

Note that you can also use the calculator to multiply 175,000 by 0.14, the decimal equivalent of 14%.

Convert the following decimals to fractions without using a calculator. Reduce them to their lowest terms.

11. 0.5 = ________
12. 0.12 = ________
13. 0.125 = ________
14. 0.8 = ________
15. 0.45 = ________

Convert the following measurements to a decimal value in feet. Round the answer to the nearest hundredth.

16. 9 inches = ________ foot
17. 10 inches = ________ foot
18. 2 inches = ________ foot
19. 4 inches = ________ foot
20. 17 inches = ________ feet

3.3.6 Practical Applications

The following are examples of some of the practical applications a tradesperson encounters daily on the job site. Use the information provided to solve each conversion problem. Be sure to show all of your work.

1. Find the cost of baseboard needed for the office building shown in the floor plan in *Figure 6*. The lumber company charges $1.19 per linear foot of baseboard, and there is a 12% discount. All door widths are the standard 30 inches. Add tax after you reduce for the sale cost.

   Costs:

   a. Total linear feet needed ________
   b. Initial baseboard cost $ ________
   c. Discount amount $ ________
   d. Baseboard cost after sale reduction $ ________
   e. Amount added for 6% tax $ ________
   f. Total baseboard cost $ ________

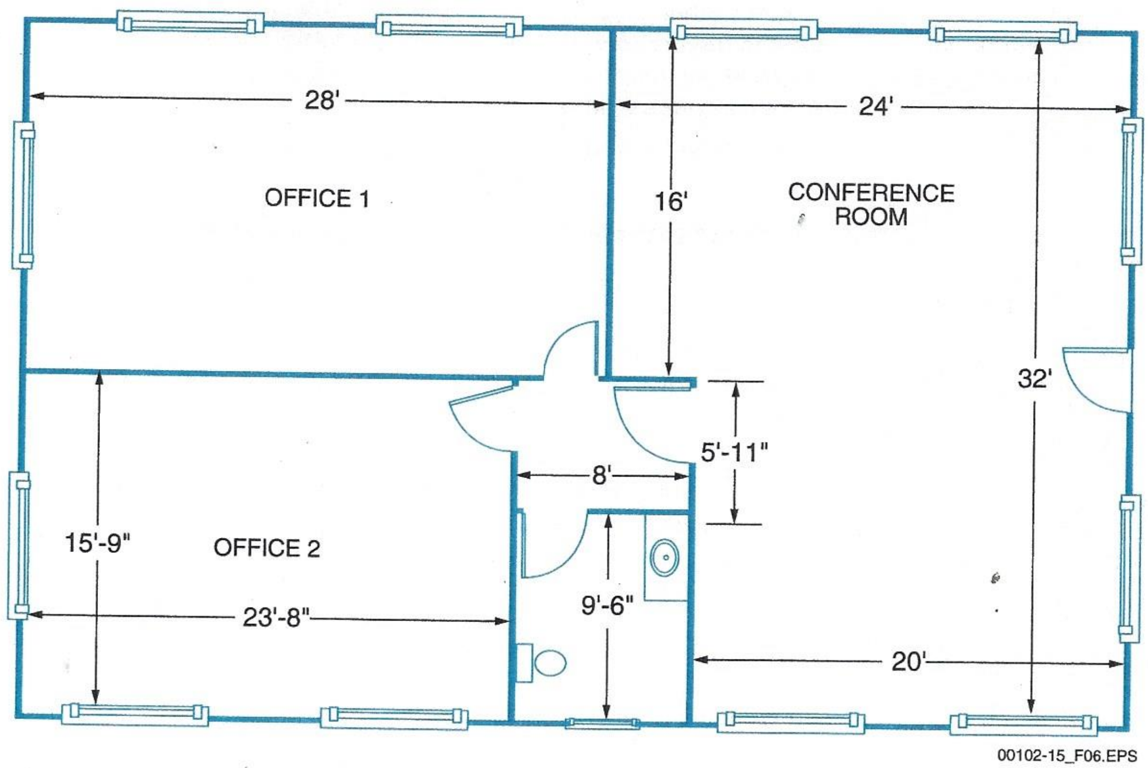

Figure 6 Office floor plan.

2. Use the site plan in *Figure 7* to determine the percentage of the lot that will be used for parking, the building, and for walkways. You may use a calculator. Round the percentages off to the nearest tenth of a percent.

   a. Percentage for parking ________ %

   b. Percentage for building ________ %

   c. Percentage for walkways ________ %

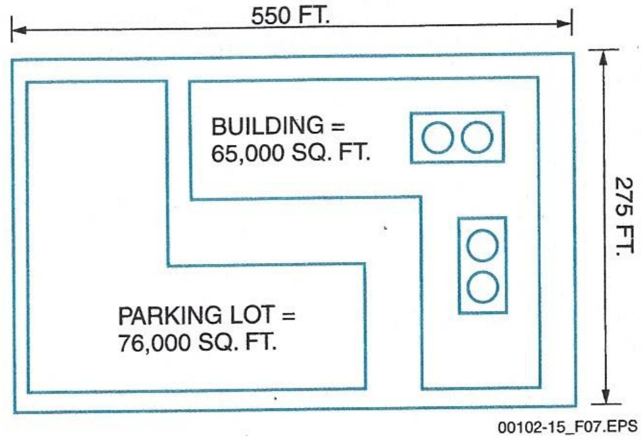

Figure 7 Site plan.

Did You Know?

Direct and Indirect Costs

When contractors determine a percentage of profit, they must take into consideration many different costs. These costs can be grouped into one of two categories: direct costs and indirect costs.

Direct costs are expenses that can be directly attributed to completing the job. For example, all of the materials (paint, insulation, lumber) and labor hours required to build a house would be considered direct costs. On larger-scale jobs, direct costs can include not only materials and labor hours, but also subcontracted demolition and cleanup crews, cranes and earth-moving machinery, or even job-site security.

Indirect costs (or overhead costs), on the other hand, are expenses that cannot be directly related to building that specific house, but are required to run the company on a day-to-day basis. Indirect costs can include such items as insurance, administrative staff payroll, marketing, office supplies, and property taxes. What other expenses can you think of that would be considered indirect costs?

GOING GREEN

City Governments Thinking Green

Governing bodies in a number of cities around the world are coming up with innovative new ways to cut energy costs and lower greenhouse-gas emissions. Some of these new ideas include creating energy by harnessing the power of waves, air conditioning a building using rooftop gardens and lake water, and integrating wind turbines into a building's construction to generate energy.

New York

In 2006, New York City Mayor Michael Bloomberg announced a plan that would cut the city's greenhouse-gas emissions by 30% by the year 2030, and would generate enough new clean energy to provide 640,000 homes with electricity. The plan outlined how bladed turbines would be submerged into New York's East River to generate power using the energy of the tidal currents to spin turbines. In 2007, the first five turbines were installed 30 feet (9.14 meters) below the river's surface.

The pilot project was such a success that city officials are installing 30 more turbines in the East River. These additional turbines should generate roughly 1,050 kilowatts (kW) of electricity— enough to satisfy the needs of 9,500 residents. The additional turbines should be fully installed in 2015.

Aspen, CO

Many years ago, Aspen became the first municipality located west of the Mississippi River to make use of hydroelectric power—a renewable energy resource. Today, over 75% of Aspen's energy comes from renewable resources. The city is trying to achieve the goal of deriving 100% of its power needs from renewable resources. Those resources include geothermal energy, hydroelectric power, solar energy, and wind. As of 2014, wind power alone was already providing 26% of the city's energy needs.

Additional Resources

Applied Construction Math: A Novel Approach, NCCER. 2006. Upper Saddle River, NJ: Prentice Hall.

Mathematics for Carpentry and the Construction Trades, Alfred P. Webster; Kathryn B. Judy. 2001. Upper Saddle River, NJ: Prentice Hall.

Mathematics for the Trades: A Guided Approach, Robert A. Carman; Hal Saunders. 2014. Pearson Learning.

3.0.0 Section Review

1. In the number 135.792, what value is represented by the 9?
   a. 90
   b. nine tenths
   c. nine one-hundredths
   d. nine one-thousandths

2. The number 0.960 is larger than the number 0.0962.
   a. True
   b. False

3. $3.625 + 4.9 =$ _____.
   a. 7.525
   b. 8.525
   c. 41.15
   d. 52.63

4. $42.58 - 7.577 =$ _____.
   a. 35.003
   b. 35.523
   c. 36.523
   d. 50.157

5. $9.64 \times 12 =$ _____.
   a. 11.568
   b. 21.64
   c. 115.2
   d. 115.68

6. $123.82 \div 6.5 =$ _____.
   a. 19.049
   b. 18.76
   c. 1.905
   d. 0.190

7. Express the number 0.479 as a percentage.
   a. 0.00479%
   b. 0.479%
   c. 47.9%
   d. 479%

8. The decimal equivalent of the fraction $\frac{7}{8}$ is _____.
   a. 0.0875
   b. 0.75
   c. 0.875
   d. 8.75

9. If a fuel tank is $\frac{2}{3}$ full, then it is more than 65% full.
   a. True
   b. False

SECTION FOUR

4.0.0 MEASURING LENGTH

Objective

Identify various tools used to measure length and show how they are used.

a. Identify and demonstrate how to use rulers.
b. Identify and demonstrate how to use measuring tapes.

Trade Terms

Joists: Lengths of wood or steel that usually support floors, ceiling, or a roof. Roof joists will be at the same angle as the roof itself, while floor and ceiling joists are usually horizontal.

Loadbearing: Carrying a significant amount of weight and/or providing necessary structural support. A loadbearing wall typically carries some portion of the roof weight and cannot be removed without risking structural failure or collapse.

Stud: A vertical support inside the wall of a structure to which the wall finish material is attached. The base of a stud rests on a horizontal baseplate, and a horizontal cap plate rests on top of a series of studs.

In the construction trade, you will need to use a measuring tool to measure the dimensions of various objects. The primary measuring devices you will see on the job are the standard English tape measure or ruler, and the metric tape measure or ruler (*Figure 8*).

CAUTION

Most rulers, especially wooden ones (similar to the metric ruler shown in *Figure 8*) are designed with extra material at the ends in order to maintain accurate measurements in the event that one of the ends is damaged. When using this type of ruler, make sure that you start your measurement at the first marked line and not at the physical end of the ruler.

4.1.0 Reading English and Metric Rulers

Reading English and metric rulers is not difficult. However, those who do not use them often may fail to recall the value of the small marks between the major units. When a measurement is misread, the error can lead to significant mistakes, which can be costly when material is scarce or expensive.

4.1.1 The English Ruler

The English ruler is divided into whole inches and then halves, fourths, eighths, and sixteenths. Some standard rulers may be divided into thirty-seconds, and some into sixty-fourths. These represent fractions of an inch. In this subsection, you will work with a standard ruler and standard fractions. In *Figure 8*, the English tape measure is marked with $\frac{1}{8}$-inch increments along the top and $\frac{1}{16}$-inch increments along the bottom. It is not uncommon to see tape measures or rulers with different markings along the top and bottom. However, the distances on the ruler shown in *Figure 9* are marked only in $\frac{1}{16}$-inch increments.

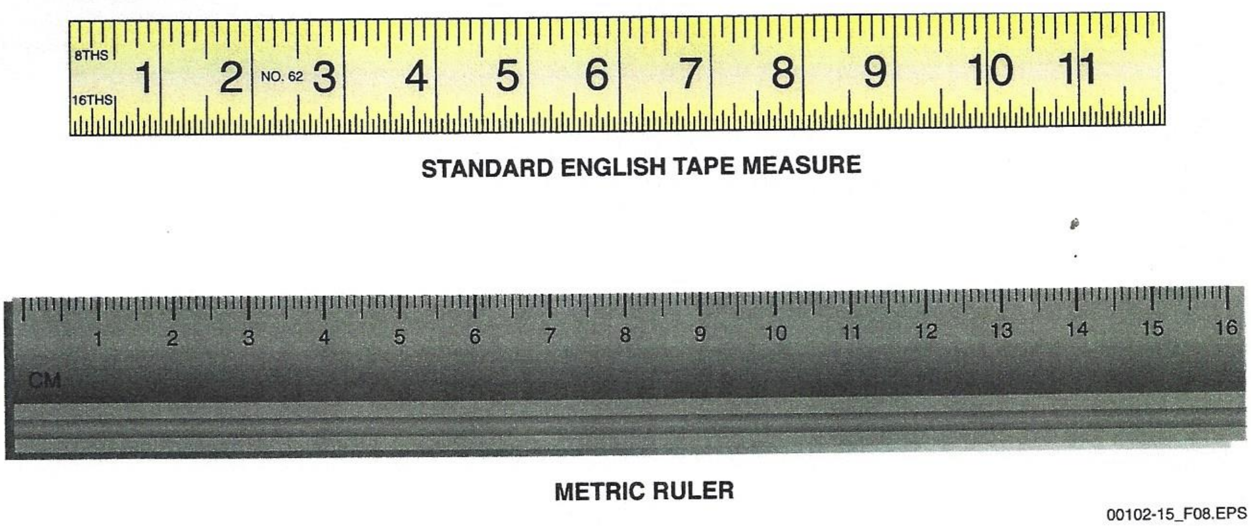

Figure 8 Various measurement tools.

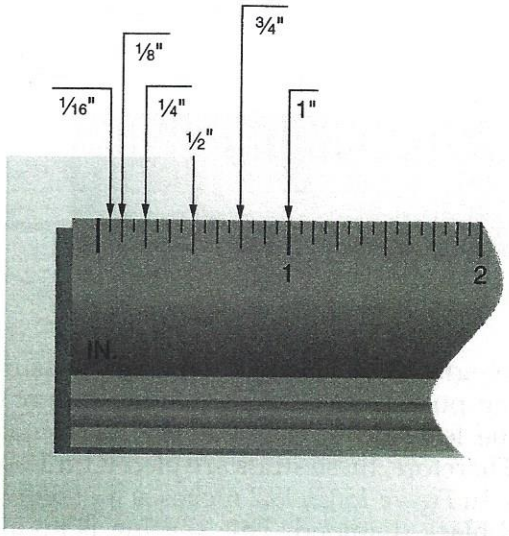

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Figure 9 Standard ruler showing 1/16-inch increments and larger values.

In *Figure 9,* the increment between the 1/8-inch and 1/4-inch increments would be called out as three-sixteenths (3/16) of an inch. Similarly, the increment immediately after the 3/4-inch increment would be called out as thirteen-sixteenths (13/16) of an inch.

4.1.2 The Metric Ruler

Metric tape measures and rulers (*Figure 10*) are typically divided into centimeters and millimeters. The larger lines with numbers printed next to them are centimeters, and the smaller lines represent millimeters. The metric system is known as a base ten system, since each millimeter increment is 1/10 of a centimeter. Therefore, if you measure 7 marks after 2 centimeters as shown in *Figure 10,* it is 2.7 (two point seven) centimeters or 27 millimeters. Both of these numbers represent the same distance.

A measurement less than 1 centimeter, six marks before the 1-centimeter mark for example, would be recorded as 0.4 (point four) centimeters or 4 millimeters. This is also shown in *Figure 10.*

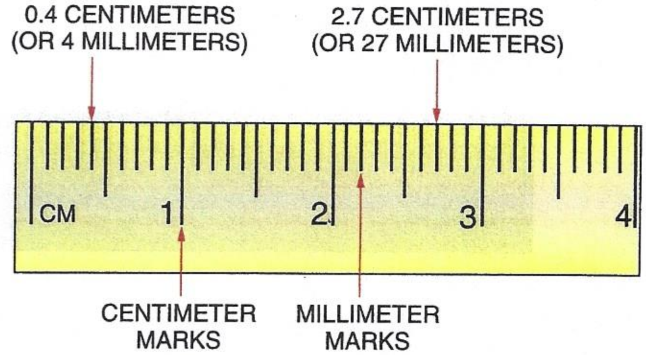

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Figure 10 Increments on a metric ruler.

4.1.3 Study Problems: Reading Rulers

Use the information from *Figure 9,* if necessary, to help you identify the marked lengths numbered 1 through 10 in *Figures 11* and *12.* Label each length in the space provided or on a separate sheet of paper.

Identify the marked lengths in *Figure 13.* Record the correct answers for Questions 11, 12, and 13 in centimeters, and the increments for Questions 14 and 15 in millimeters.

4.2.0 The Measuring Tape

Measuring tapes are commonly referred to as tape measures; both names are quite common, depending upon the area of the country or world. The measuring tape's blades show either Imperial markings, metric markings, or possibly both as shown in *Figure 14.* If the blade only has markings from one system, which is most common, they are printed along both edges of the blade so that measurements can be taken accurately from either side.

Some common lengths for measuring tapes using both measurement standards are 16 feet (5 meters) and 25 feet (8 meters). There are many measuring tape lengths available with only one set of markings on the blade. Metric-only tapes are commonly available in 3.5, 5, and 8 meter lengths, for example. Longer tapes are generally

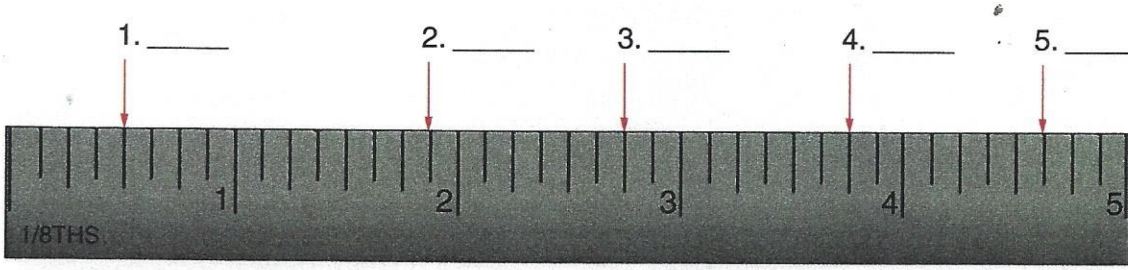

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Figure 11 Practice with a 1/8-inch ruler.

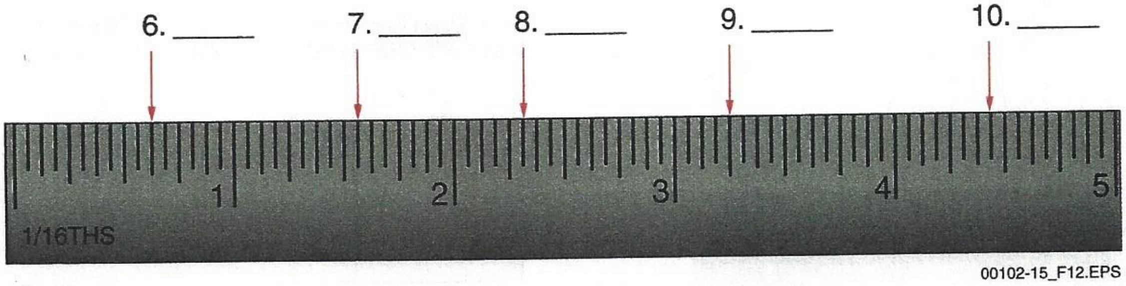

Figure 12 Practice with a 1⁄16-inch ruler.

used for site layout, while shorter tapes are more compact and convenient for carpentry, pipefitting, and other crafts.

4.2.1 The English Measuring Tape

The English measuring tape is marked similarly to the ruler described earlier with a few additional markings. Along with the ½-inch, ¼-inch, ⅛-inch, and 1⁄16-inch markings, a measuring tape usually features additional markings that make the task of wall framing easier.

As shown in *Figure 15(A)*, every 24 inches is marked with a contrasting black background. Note that 24-inch spacing on center is used most commonly for the **studs** in walls that do not carry a heavy load from above. The markings in *Figure 15(B)* are used for the 16-inch on center spacing most commonly used for **loadbearing** walls. These are highlighted with a red background. This common stud spacing is shown in the photograph in *Figure 15*. With the measuring tape stop pulled tight against the left side of one stud, the left side of the next stud is 16 inches away. Therefore, these studs are placed on 16-inch centers. In *Figure 15(C)*, 19.2 inches is marked with a small black diamond. This spacing is an alternate and less commonly used spacing scheme for special **joists**. It is not generally used for stud spacing.

Figure 16 shows other markings that may be printed on a standard tape measure. The red foot number (1F) works in conjunction with the red inch number (2) to quickly determine a measurement. The measurement indicated is 1 foot, 2 inches. The number below it (black 14) indicates 14 inches. If you add the top scale numbers together, it will equal the number of inches displayed on the bottom scale (1 foot or 12 inches + 2 inches = 14 inches). The red numbers on the top are marked in the same way along the length of the blade.

Around the World

Nominal and True Dimensions

A 2" × 4" board used in the United States is not actually 2 inches by 4 inches in dimension. The only time the board is truly 2 inches by 4 inches is when it is initially rough-cut from the log. After the board has been dried and planed, it is reduced to a finished size of 1½ inches by 3½ inches. Here are some true dimensions of other common board sizes in the Imperial system:

Nominal Size	Actual Measure	Actual Metric Measure
1" × 10"	¾" × 9¼"	19 × 235 mm
2" × 6"	1½" × 5½"	38 × 140 mm
2" × 8"	1½" × 7¼"	38 × 184 mm
2" × 10"	1½" × 9¼"	38 × 235 mm
4" × 4"	3½" × 3½"	89 × 89 mm

Equivalent lumber sizes in the metric world were generally converted using 25 mm per inch as a base. Note that this is not the precise value of an inch in the metric system, but this value was chosen for simplicity. As a result, the equivalent of a 2" × 4" in the metric system is referred to as a 50 mm × 100 mm board. However, as is the case in the United States, the actual dimensions are smaller; typically 40 mm × 90 mm. Over the years, the actual size of lumber compared to its nominal size has steadily decreased. Continued reductions in lumber sizes may result in changes to structural codes to maintain building strength.

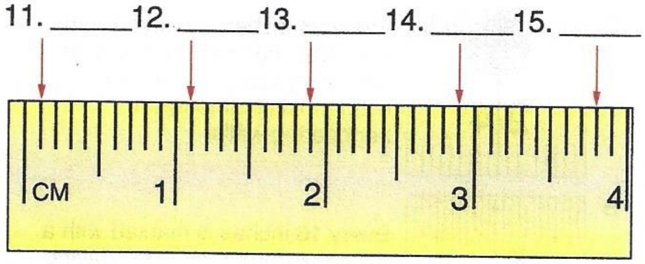

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Figure 13 Metric ruler practice.

4.2.2 The Metric Measuring Tape

Metric measuring tapes are made in the same basic way as a standard measuring tape. In the United States, it may be difficult to find a measuring tape that uses only metric increments. At best, they will have both systems of measurement marked on the tape. In other countries, metric measuring tapes without inch markings on them are the most common.

Most metric-only measuring tapes do not have the same markings for standard stud spacing and other common spacing values. When they do, the markings are different to accommodate national or regional standards and the common dimensions of metric lumber.

4.2.3 Using a Tape Measure

Using a tape measure is relatively simple. You will note that the tape is not flat, but instead is concave on the top. This strengthens the tape and allows it to be extended a significant distance outside the case while remaining straight, without support along its length. Thicker blade material and a more concave shape allows for longer extension. In marketing literature, manufacturers may refer to this distance as stickout.

Step 1 Pull the tape straight out of its case with one hand.

Step 2 Hook the tape to one end of the object being measured using the metal hook on the end of the tape. If you are unable to hook onto the material, it may be necessary to have another worker hold the end of the tape. Be aware that the hook end is designed to slide a small amount to compensate for its own thickness; it is slightly loose and free to move for a very good reason. It slides in slightly when making an inside measurement, and slides out when the hook is used to make an outside measurement. If the end does not slide freely, your measurements can be off by as much as $\frac{1}{16}$ (1.6 mm) of an inch. A bent or damaged hook can have the same effect.

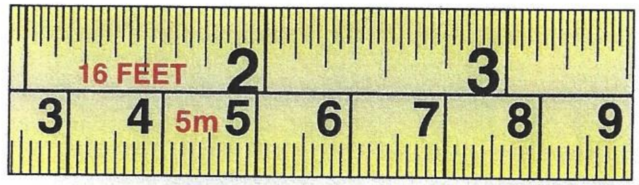

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Figure 14 Measuring tape showing Imperial and metric measurements.

Step 3 Extend the tape by pulling the case, allowing the tape to extend to the point where the measurement will be taken.

Step 4 When the desired tape length is reached, slide the thumb lock on the case down to hold the tape in place. Not all tape measures have a thumb lock, and some tape measures will lock automatically. On automatic locking tape measures, a thumb hold release is used to retract the tape. Never let the tape retract rapidly back into the case, because this can cause damage to the hook.

It is sometimes necessary to use a tape measure to make an inside measurement. For example, you may need to make a measurement between two walls to determine the length of a shelf. Most measuring tapes have a small notation that indicates the exact size of the case. By firmly bumping the hook against one wall and the back of the case against the other wall, a measurement can be taken. Read the length where the tape meets the case, and then add the length of the case as printed or stamped on it.

Number of Courses

When erecting block walls, you will often hear the term *number of courses.* The number of courses refers to the number of block rows that will need to be stacked in order to establish the correct height of the wall. If you are told to erect a 15-course wall using 8" × 8" × 16" concrete blocks, you would be building a wall that is 10 feet high.

Block height = 8"
Number of courses = 15
12 inches = 1 foot

Therefore:

8" × 15 = 120"
120" ÷ 12" = 10 feet

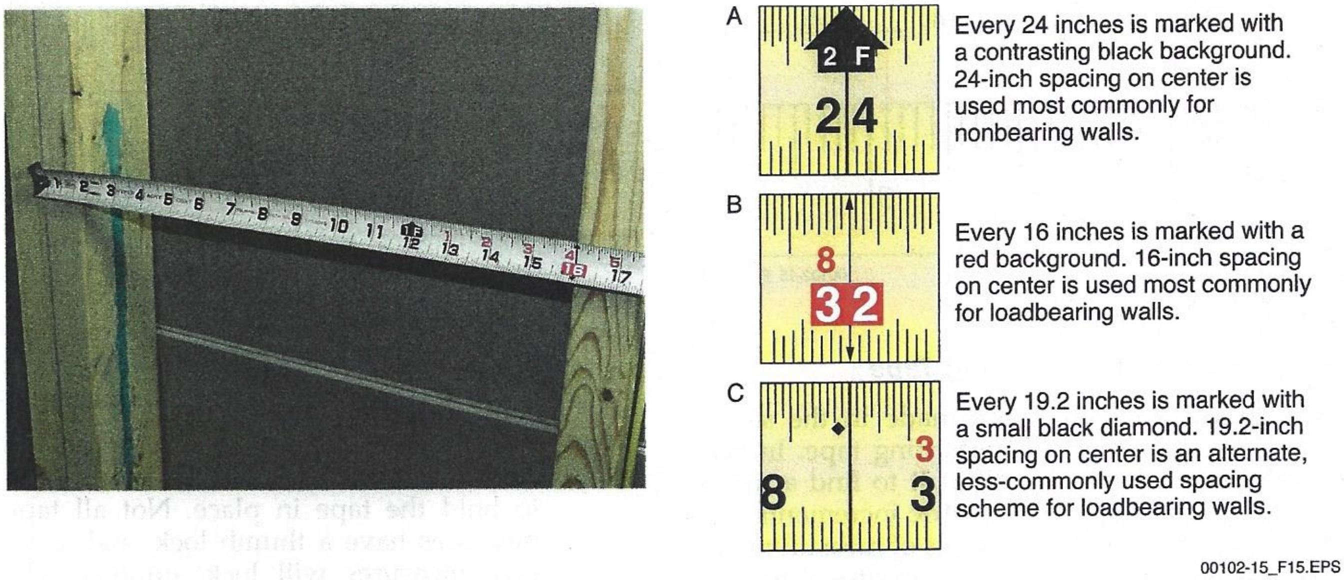

Figure 15 Wall-framing markings on a tape measure.

> **NOTE**
> Always remember to double-check your measurements before cutting a piece of material. Keep in mind the old saying, "Measure twice, cut once."

4.2.4 Study Problems: Reading Measuring Tapes

Identify the marked lengths numbered 1 through 5 in *Figures 17* and *18*. Record your answers on a separate piece of paper. Note that answers related to *Figure 17* should be in inches (using fractions as necessary). The answers related to *Figure 18* should be in centimeters (using tenths of a centimeter as necessary).

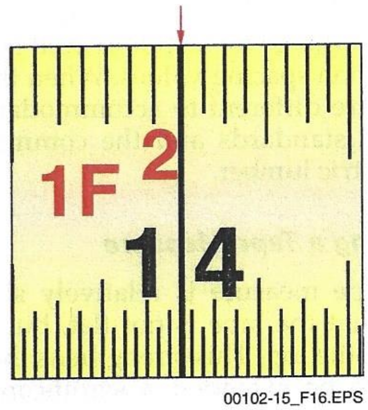

Figure 16 Other markings on a standard tape measure.

Around the World

Visualizing Metric Units

For users of the inch-pound system, it may help to visualize some metric measurements, using the examples below.

Metric Examples Visualized (approximation only):

- 1 millimeter = the thickness of the edge of a dime
- 1 centimeter = the width of a standard paperclip
- 1 decimeter = the length of a crayon
- 1 meter = the distance from a door handle to the floor (about 1.1 yards)
- 1 kilometer = the length of 6 city blocks (about 0.6 miles)
- 1 gram = weight of a paperclip
- 1 kilogram = weight of a brick (about 2.2 pounds)

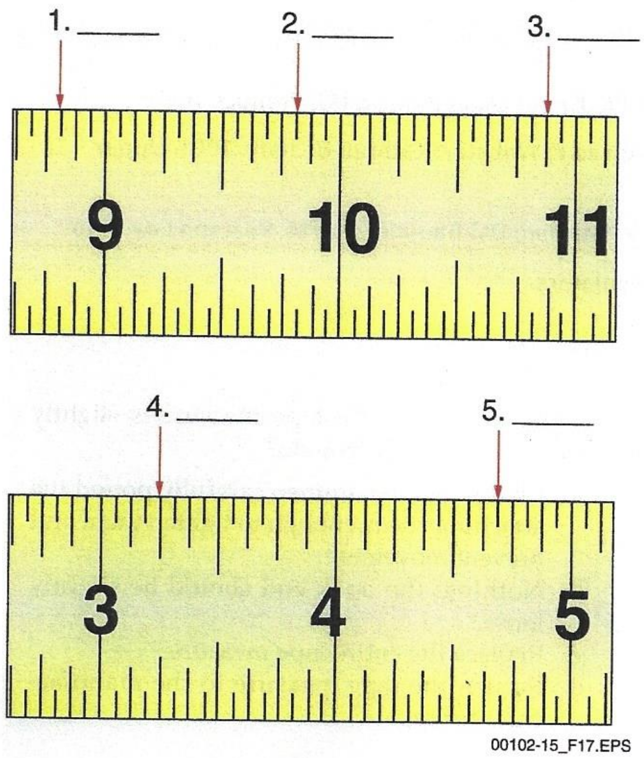

Figure 17 Tape measure practice.

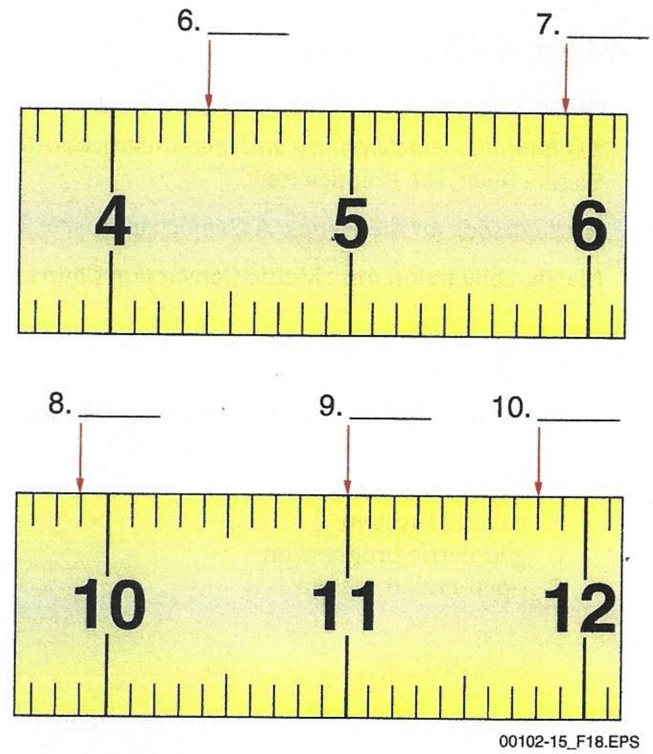

Figure 18 Metric tape measure practice.

Did You Know?

Standard Measure

Determining the distance to a neighbor's farm may have been the earliest measurement of interest to people. Imagine that you wanted to measure the distance to a favorite fishing pond. You could walk, or pace off, the distance and count the number of steps you took. You might find that it was 570 paces between your home and the pond. You could then pace off the distance to another pond. By comparing the paces of one distance to the other, you could tell which distance was longer and by how much. Pace became a unit of measure. Of course, the length of individual paces differs from person to person.

Feet, arms, hands, and fingers were useful for measuring all sorts of things. In fact, the body was such a common basis for measuring that we still have traces of that system in our measurement standards. Lengths are measured in feet. Horses are said to be some number of hands high. An inch roughly matches the width of a person's thumb.

Complications arise when you have to decide whose legs, arms, or hands to use for measuring. If you wanted to mark off a piece of land, you might choose a tall person with long legs in hopes of getting more than your money's worth. If you owed someone a substantial amount of rope, you would want someone with short arms to measure the quantity and possibly save you some rope.

The problem demanded a standardized system of measurement that everyone could agree on. Legend says that the foot measurement used in France was the length of Charlemagne's foot. The yard is supposed to have been the distance from King Henry I of England's nose to the fingertips of his outstretched arm. Kings were not going to travel around making measurements for people, so the solution was to transfer the yard measurement to a stick. The distance marked on the stick became the standard measurement, and the government could send out duplicate sticks to each of the towns and cities as secondary standards. Civil officials could then check local merchants' measurements using the secondary standards.

During the Middle Ages, associations of craftspeople enforced strict adherence to the established standards. The success and reputation of the craft depended on providing accurately measured goods. Violators who used faulty measurements were often punished.

Additional Resources

Applied Construction Math: A Novel Approach, NCCER. 2006. Upper Saddle River, NJ: Prentice Hall.

Mathematics for Carpentry and the Construction Trades, Alfred P. Webster; Kathryn B. Judy. 2001. Upper Saddle River, NJ: Prentice Hall.

Mathematics for the Trades: A Guided Approach, Robert A. Carman; Hal Saunders. 2014. Pearson Learning.

Metric-conversion.org : Metric Conversion Charts and Calculators.

4.0.0 Section Review

1. The metric system can be referred to as a(n) _____.
   a. base 10 system
   b. base 100 system
   c. geometric progression
   d. open-ended system

2. If the hook end of a tape measure is slightly loose, what should you do?
   a. Using a small hammer, carefully pound the attaching rivets enough to tighten them and prevent movement.
   b. Nothing; the hook end should be slightly loose.
   c. Replace the entire tape measure.
   d. Return the tape measure to the manufacturer for repair.

SECTION FIVE

5.0.0 METRIC AND IMPERIAL MEASUREMENT SYSTEMS

Objective

Identify and convert units of length, weight, volume, and temperature between Imperial and metric systems of measurement.

a. Identify and convert units of length measurement between the Imperial and metric systems.
b. Identify and convert units of weight measurement between the Imperial and metric systems.
c. Identify and convert units of volume measurement between the Imperial and metric systems.
d. Identify and convert units of temperature measurement between the Imperial and metric systems.

Trade Terms

Force: A push or pull on a surface. In this module, force is considered to be the weight of an object or fluid. This is a common approximation.

Mass: The quantity of matter present.

Unit: A definite standard of measure.

Volume: The amount of space contained in a given three-dimensional shape.

More than 95-percent of the world uses the metric system of measurement. Products are manufactured across the world and shipped to global destinations on a daily basis. The dimensions, weights, temperatures, and pressures related to these products may be provided using metric system values, Imperial system values, or both.

The use of the metric system in the United States is becoming more common. Many US manufacturers publish their documentation using values from both systems, regardless of the product destination. For some trades, it is essential to become familiar with both systems and understand how to convert from one to the other. *Figure 19* shows some metric system and Imperial system values that have become familiar to many people through the marketplace.

A great deal of work in science, engineering, and the trades is based on the exact measurement of physical quantities. A measurement is simply a comparison of a quantity to some definite standard measure of dimension called a **unit**. Whenever a physical quantity is described, the units of the standard to which the quantity was compared, such as a foot, a liter, or a pound, must be specified. A number alone is not enough to describe a physical quantity.

Once it is understood, the metric system is actually simpler to use than the Imperial system. This is because it is a decimal-based system in which unit prefixes are used to denote powers of ten. The Imperial system, on the other hand, requires the use of conversion factors from one Imperial unit to another. These factors must be memorized or determined from tables. For example, one mile is 5,280 feet, and 1 inch is 1/12 of a foot. In contrast, a centimeter is one-one hundredth of a meter and a kilometer is 1,000 meters. Any conversion done within the metric system of measurement involves some power of 10. An additional advantage of the metric system is that it is not necessary to add and subtract fractions for measurement purposes.

Metric system prefixes are listed in *Table 1*. From this table, it can be seen that the metric system is logically arranged, and that the prefix of the unit represents its order of magnitude.

The most common metric system prefixes are mega- (M), kilo- (k), centi- (c), milli- (m), and micro- (μ). Even though these prefixes may seem difficult to understand at first, many people are probably more familiar with them than they realize.

5.1.0 Units of Length Measurement

Sometimes you may need to change from one unit of measurement to another within the same system—for example, from inches to yards or from centimeters to meters. You may also need to convert length measurements from one system to the other. Before considering conversions between the two systems of measurement, we will examine the common units of length in each system.

5.1.1 Imperial System Units of Length

Table 2 shows the most common units of length in the Imperial system and their relationships. The inch is often broken down into fractions, as was presented in the section related to rulers and measuring tapes. As a general rule, the smallest

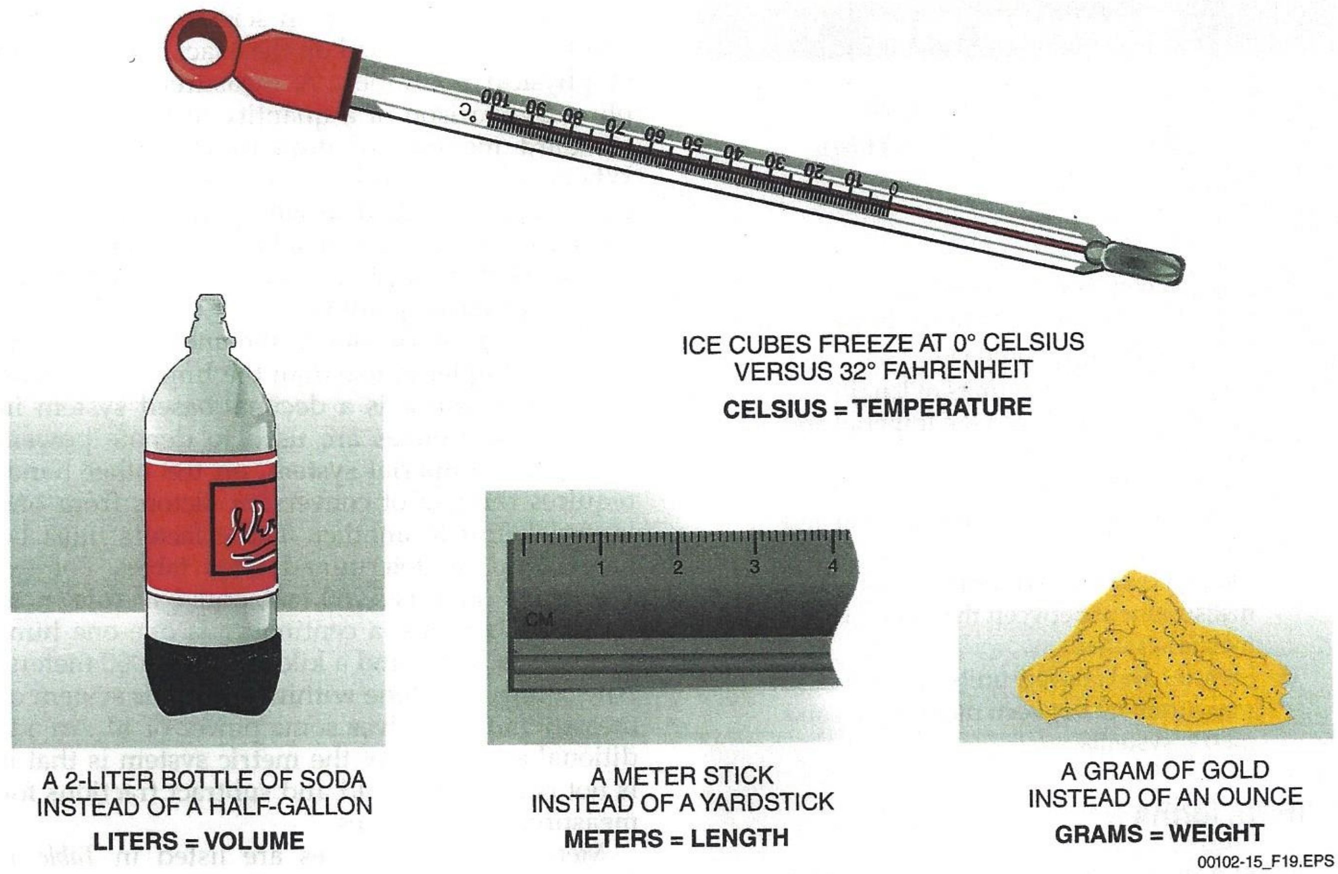

Figure 19 Common Imperial and metric measured values.

fractional value the inch is broken into is 1/64. For work requiring an even higher level of precision, the inch can also be broken down to decimal values. The increments are usually thousandths (0.001 inches) or ten-thousandths (0.0001 inches) when extreme precision is required.

Each of the units shown here may be abbreviated several different ways:

- An inch or inches may be abbreviated as *in* or identified by the symbol ".
- A foot or feet may be abbreviated as *ft* or identified by the symbol '.
- A yard or yards may be abbreviated as *yd*.
- A mile or miles is rarely abbreviated, but the most likely abbreviation is *mi*.

Converting one unit to another involves multiplication or division by the proper value. This can sometimes be done in steps, but it is usually best to do it in one step when a calculator is available. For example, to change from inches to yards, you may first divide the number of inches by 12 (the number of inches in a foot) to find the number of feet. You would then divide that number by 3 (the number of feet in a yard) to find the number of yards. This method may be simpler when trying to make the conversion without pencil and paper or a calculator. A quicker way though, is to divide the number of inches by 36. (*Figure 20*). This requires less work and eliminates a step, which reduces the opportunity for an error.

5.1.2 Metric System Units of Length

Table 3 shows some common units of length used in the metric system and their relationships. Notice again that all the units are related to each other by some power of ten.

These units may be abbreviated as follows:

- A millimeter is abbreviated as *mm*.
- The centimeter is abbreviated as *cm*.
- The meter is abbreviated as *m*.
- The kilometer is abbreviated as *km*.

Conversions within the metric system are easier. To make a conversion, you simply move the decimal point, because the system is based on multiples of 10. The keys to success are knowing which direction to move the decimal point, and how far. When converting from a smaller value to a larger one, the decimal point must move to the left. When converting from a larger value to a smaller one, the decimal point must move to the right. Tables and charts provide the information to determine how far to move the decimal point.

Table 1 Metric System Prefixes

Prefix		Unit		
micro- (μ)	1/1,000,000	0.000001	10^{-6}	One-millionth
milli- (m)	1/1,000	0.001	10^{-3}	One-thousandth
centi- (c)	1/100	0.01	10^{-2}	One-hundredth
deci- (d)	1/10	0.1	10^{-1}	One-tenth
deka- (da)	10	10.0	10^{1}	Tens
hecto- (h)	100	100.0	10^{2}	Hundreds
kilo- (k)	1,000	1,000.0	10^{3}	Thousands
mega- (M)	1,000,000	1,000,000.0	10^{6}	Millions
giga- (G)	1,000,000,000	1,000,000,000.0	10^{9}	Billions

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With the above in mind, determine how many meters there are in 72 centimeters (*Figure 21*). Since 1 centimeter = 0.01 meter, the decimal point must move two positions. Further, since a smaller value is being converted to a larger one, the decimal point must move to the left. Therefore, 72 centimeters becomes 0.72 meters.

It is important to note that moving decimal points provides the same result as multiplication, but without the math. For example, 72 centimeters can also be converted to meters by dividing by 100, since 1 meter is equal to 100 centimeters. The result remains the same at 0.72 meters.

Table 2 Common Imperial Units of Length

IMPERIAL LENGTH UNITS		
1 inch	=	1/12th of a foot; 0.0833 feet
1 foot	=	12 inches; 1/3rd of a yard
1 yard	=	36 inches; 3 feet
1 mile	=	5,280 feet; 1,760 yards

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5.1.3 Converting Length Units Between Systems

Converting measurements from the Imperial system to the metric system, and vice versa, is much like converting units within the Imperial system; multiplication or division by some factor is required. Since both systems are not based on powers of 10, moving the decimal point alone will not work. Many reference books, including dictionaries, contain charts or tables that show basic equivalents between the Imperial system and metric system units. Examples of some comparison charts are provided in the *Appendix*.

The successful conversion of units from one system to another depends heavily on using the proper conversion value. Even if the mathematical calculation is correct, using the wrong factor will result in an incorrect result. *Table 4* provides some common factors for converting length measurements from one system to another. Once the math is done and the new value has been determined, you can determine if the number can be rounded off to some degree.

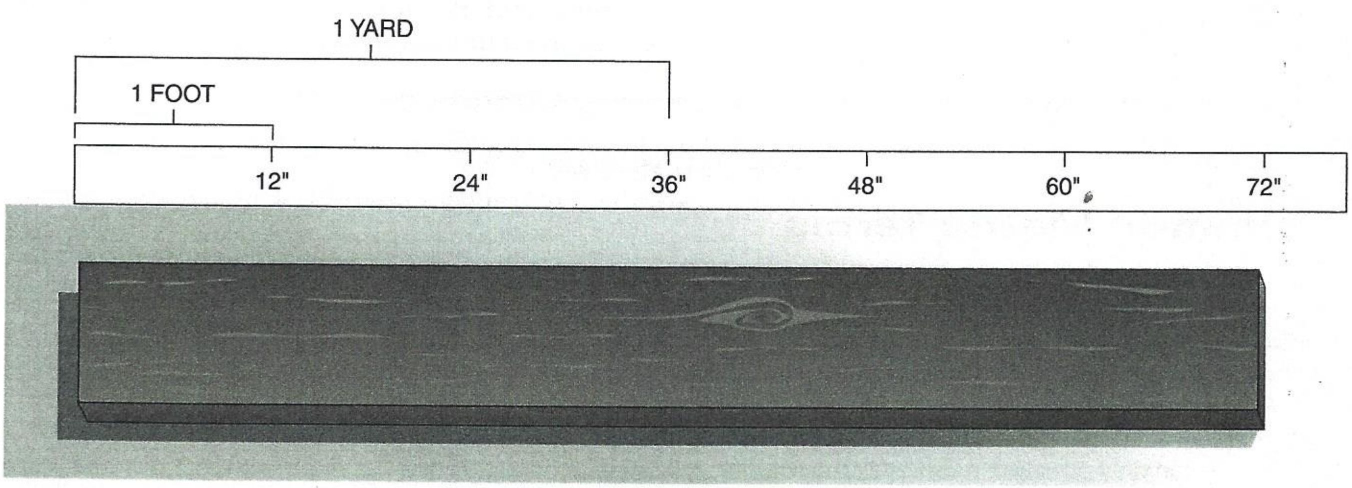

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Figure 20 Converting inches to yards.

Table 3 Common Metric Units of Length

METRIC LENGTH UNITS		
1 kilometer	=	1,000 meters
1 meter	=	100 centimeters; 0.001 kilometers
1 centimeter	=	10 millimeters; 0.01 meters
1 millimeter	=	0.1 centimeters; 0.001 meters

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For example, according to *Table 4*, 1 centimeter is equal to 0.3937 inches. To convert 13 centimeters to inches, multiply this factor times 13. The result is 5.1181 inches. For most applications, this number can be rounded to 5.12 inches, or even to 5.1 inches. The degree of rounding is dependent upon the level of accuracy required in the measurement. If, for instance, the conversion is related to determining the precise center of a steel plate, then a high level of accuracy may be in order. However, if the conversion is related to measuring how far away from a driveway to set a mailbox post, then a high level of accuracy is far less important.

5.1.4 Study Problems: Converting Measurements

Find the answers to the following conversion problems without using a calculator. Round your answers to the nearest hundredth.

1. 0.45 meter = ________ centimeters
2. 3 yards = ________ inches
3. 36 feet = ________ yards
4. 90 inches = ________ yards
5. 1 centimeter = ________ meters
6. 66 inches = ________ centimeters

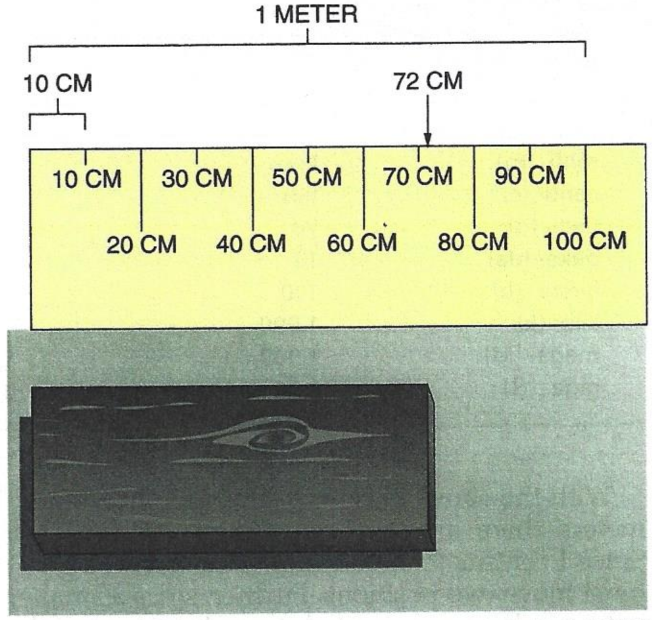

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Figure 21 Converting centimeters to meters.

7. 47 feet = ________ meters
8. 54.5 centimeters = ________ feet
9. 19 yards = ________ meters
10. 4.7 meters = ________ inches

5.2.0 Units of Weight Measurement

This section will focus on units used to measure weight in both the Imperial and metric systems.

5.2.1 Imperial Units of Weight

Weight is actually the **force** an object exerts on the surface of the Earth due to its **mass** and the pull of the Earth's gravity. In the Imperial system, common units for weight include the ounce, the pound, and the ton. The relationship between these three units is shown in *Table 5*.

Did You Know?

Common Metric Terms

The most common units of length measure in the metric system are the millimeter, centimeter, meter, and kilometer. Unit names for measurements larger than a kilometer are not commonly used. For example, the term *hectometer* (100 meters) is not often used. In the Olympics, you hear about the 200-meter and the 400-meter races, not the 2-hectometer and the 4-hectometer races.

Measurements smaller than the millimeter (0.001 meter) are usually used by scientists and in precision machining operations. The micrometer (0.000001 meter), nanometer (0.000000001), and the picometer (0.000000000001 meter) are certainly not used in everyday measuring.

Table 4 Length Conversion Factors

Unit	Centimeter	Inch	Foot	Meter	Kilometer
1 millimeter	0.1	0.03937	0.003281	0.001	0.000001
1 centimeter	1	0.3937	0.3281	0.01	0.00001
1 inch	2.54	1	0.08333	0.0254	0.0000254
1 foot	30.48	12	1	0.3048	0.0003048
1 meter	100	39.37	3.281	1	0.001
1 kilometer	100,000	39,370	3,281	1,000	1

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Table 5 Common Imperial Units of Weight

IMPERIAL WEIGHT UNITS		
1 ounce	=	1⁄16th of a pound; 0.0625 pounds
1 pound	=	16 ounces; 0.0005 tons
1 ton	=	2,000 pounds

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These units of measure may be abbreviated as follows:

- An ounce or ounces may be abbreviated as *oz* or *ozs.*
- A pound or pounds may be abbreviated as *lb* or *lbs.*
- The ton may be abbreviated as *t.*

Conversions from one weight unit to another within the Imperial system are done in the same manner as length conversions. For example, to convert 134 ounces to pounds, divide by 16—the number of ounces in one pound. The result is 8.375 pounds. The number can be rounded depending upon the level of precision required for the result.

5.2.2 Metric Units of Weight

The most commonly used metric units of weight are the milligram, the gram, and the kilogram. As is the case with length, these metric units are related by powers of ten. Their relationships are shown in *Table 6.*

Use the following abbreviations with these units:

- A milligram is abbreviated as *mg.*
- The gram is abbreviated as *g.*
- The kilogram is abbreviated as *kg.*

Converting weight units within the metric system can be done through multiplication, or by simply moving the decimal point the proper

Around the World

Metric System as a Modern Standard

As scientific thought developed during the 1500s and later, scholars and scientists had trouble explaining their measurements to one another. The measuring standard varied among the different cultures. By the 1700s, scientists were debating how to establish a uniform system for measurement.

In the 1790s, French scientists created a standard length called a meter (based on the Latin word for measure). This became the basis for the metric system that is still in use throughout most of the world. Not until the 1970s did both the United States and Canada begin to truly acknowledge the metric system. This was partly driven by massive increases in global trade. There is still resistance to fully converting to the metric system in the United States, even though virtually every other country in the world uses it.

The original international standard meter—a solid bar measuring precisely a meter—was made of platinum-iridium and kept in the International Bureau of Weights and Measures near Paris, France. The bar was made from a platinum-iridium alloy because the material would not rust or change over time. That ensured the accuracy of the standard.

In more modern times, scientists have found that a natural standard of measure was more accurate than anything made out of metal. In 1983 the speed of light was calculated as precisely 299,792,458 meters per second. So the distance that light travels in 1/299,792,458 second is now the standard for a meter.

Table 6 Common Metric Units of Weight

METRIC WEIGHT UNITS		
1 metric ton	=	1,000 kilograms
1 kilogram	=	1,000 grams; 0.001 metric tons
1 gram	=	1,000 milligrams; 0.001 kilograms
1 milligram	=	0.001 gram; 0.000001 kilograms

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Water and the Gram

In the metric system, the milliliter is a popular unit of liquid measure, equal to 1⁄1000th (0.001) of a liter. One milliliter of pure water weighs exactly one gram. This is yet another example of the practicality of the metric system. Other liquids, of course, will have different weights, depending on their density.

number of places in the correct direction. For example, *Table 6* shows that 1 gram is equal to 1,000 milligrams. To convert 6,439 milligrams to grams, divide 6,439 grams by 1,000—the number of milligrams in a single gram. The alternative is to move the decimal point to the left three places. Moving three decimal places is correct since a milligram is 0.001 grams. Regardless of the method chosen, the result is the same—6.439 grams.

5.2.3 Converting Weight Units Between Systems

Table 7 shows the relationship of weight units between the Imperial and metric systems. The equivalent values in some cases have been carried a number of places beyond the decimal point. You may not always need to make calculations at this level of detail, depending upon the related task. The result of a conversion can also be rounded off to the level of precision required for the application.

Weight-related metric conversions might involve the weight of refrigerant for air-conditioning system charging or the amount of lubricant that must be added to an engine. There are many possible situations where weight conversion may be necessary. Imagine that you must set a new piece of equipment on a rooftop, but the specifications provide the weight in metric units as 1,200 kilograms (kg). How do you select the equipment needed to safely raise the unit onto the roof when the rigging equipment is rated in pounds only?

Table 7 shows that 1 kg = 2.205 pounds, so multiplying 1,200 kg by 2.205 yields 2,646 pounds. Therefore, any slings or lifting equipment used to raise the unit must have a rated capacity greater than 2,646 pounds.

When working with smaller weights, ounces or grams are often used as the unit of measure. One ounce is equal to 28.35 grams, so the gram is obviously the smaller unit of measure. Ounces can then be converted to grams by multiplying the number of ounces by 28.35. Grams are converted to ounces by dividing by 28.35.

5.2.4 Study Problems: Converting Weight Units

Convert these weights from imperial to metric weight units, or vice versa. Answers should be stated to the nearest hundredth.

1. 50 pounds = _______ kilograms
2. 50 kilograms = _______ pounds
3. 15.9 ounces = _______ grams
4. 94 grams = _______ ounces

Table 7 Imperial and Metric Weight Conversion Chart

IMPERIAL AND METRIC WEIGHT CONVERSION							
Unit	**Metric Ton**	**Ton**	**Kilogram**	**Pound**	**Ounce**	**Gram**	**Milligram**
1 metric ton	1	1.102	1,000	2,204.62	35,274	1,000,000	1,000,000,000
1 ton	0.9072	1	907.185	2,000	32,000	907,185	907,200,000
1 kilogram	0.001	0.0011	1	2.205	35.27	1,000	1,000,000
1 pound	0.000454	0.0005	0.4536	1	16	453.6	453,592
1 ounce	0.00002835	0.00003125	0.02835	0.0625	1	28.35	28,349.5
1 gram	0.000001	0.000001102	0.001	0.002205	0.03527	1	1,000
1 milligram	0.000000001	0.000000001102	0.000001	0.000002205	0.00003527	0.001	1

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5.3.0 Units of Volume Measurement

In this section, units of volume measurement in the Imperial and metric systems will be examined. Note that volume here applies to three-dimensional objects such as cubes and cylinders. Liquid units of measure are not included in this discussion.

Volume is the amount of space occupied by a three-dimensional object such as a barrel or the piston of an engine. Three separate two-dimensional measurements are typically needed to properly calculate volume: length, width, and height. One of these dimensions, usually height, may be referred to as depth or thickness. Cubic units of measure describe the volume of different spaces. The same units used for length measurement—such as inches or centimeters—are used in units of volume. The result of multiplying the three measurements together to determine volume is indicated by adding the word *cubic* to the unit of measure. For example, if a cube measures 1" × 1" × 1", the volume is shown as 1 cubic inch.

A review of volume calculations for various three-dimensional shapes is provided in Section Six of this module.

5.3.1 Imperial Units of Volume

Table 8 shows the three most common units of volume measurement in the Imperial system. Note that these units are all based on familiar units of length measurement.

These units can be abbreviated as follows:

- The cubic inch may be abbreviated as *cu in* or *in*^{3}. It may also be seen as CI but this is not a standard abbreviation.
- The cubic foot may be abbreviated as *cu ft* or *ft*^{3}.
- The cubic yard may be abbreviated as *cu yd* or *yd*^{3}.

The units shown here can be converted from one to the other in the same manner as other units of measure. For example, to convert 864 cubic inches to cubic feet, multiply 864 by the conversion factor of 0.0005787. The result is 0.4999, or 0.5 cubic feet.

Table 8 Common Imperial Units of Volume

IMPERIAL VOLUME UNITS		
1 cubic inch	=	0.0005787 cubic feet
1 cubic foot	=	1,728 cubic inches; 0.037 cubic yards
1 cubic yard	=	27 cubic feet

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5.3.2 Metric Units of Volume

The most common metric units used for volume are the cubic centimeter and the cubic meter. Their relationship to each other is shown in *Table 9*. Like all units of measure in the metric system, there are other volume units such as the cubic millimeter and cubic kilometer, all having a relationship to each other based on powers of ten. These two however, are the most commonly used in the trades. Therefore the conversion from one unit to another within the metric system is done in the same manner as other metric units of measure. The cubic meter represents a three-dimensional object that is 100 centimeters × 100 centimeters × 100 centimeters in size. Therefore, 1 cubic meter is equal to 1,000,000 cubic centimeters.

Cubic centimeters may be abbreviated as *cu cm* or *cm*^{3}. Cubic meters may be abbreviated as *cu m* or *m*^{3}.

5.3.3 Converting Volume Units Between Systems

Table 10 shows the relationship between Imperial and metric units of volume. Workers may need to convert volume from one system to the other when working with concrete for a foundation or a given amount of soil in site work. Only in rare cases would there be a need to convert volumes between very large units, such as cubic meters, and very small units such as cubic inches or centimeters. In most cases, the conversion will be to a unit in the other system that is relatively close to the same volume. Converting from cubic inches to cubic centimeters, or cubic yards to cubic meters, would be far more likely.

For this example, cubic centimeters will be converted to cubic inches. Assume that an enclosure must be large enough to accommodate a cube-shaped battery that the manufacturer says has a volume of 1,600 cubic centimeters. You know the area allowed for the battery is also cube-shaped and measures 112 cubic inches. Will the battery fit?

To convert cubic centimeters to cubic inches, multiply the number of cubic centimeters by the conversion factor. Since the conversion factor is 0.0610, multiply 1,600 by 0.0610. The result is 97.6 centimeters. So a cube-shaped object of 1,600 centimeters will fit into a space equal to 112 cubic inches, with room to spare.

Table 9 Common Metric Units of Volume

METRIC VOLUME UNITS		
1 cubic centimeter	=	0.000001 cubic meters
1 cubic meter	=	1,000,000 cubic centimeters

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Table 10 Imperial and Metric Volume Conversion Chart

IMPERIAL AND METRIC VOLUME CONVERSION					
Unit	**Cubic Meter**	**Cubic Yard**	**Cubic Foot**	**Cubic Inch**	**Cubic Centimeter**
1 cubic meter	1	1.308	35.315	61,023.7	1,000,000
1 cubic yard	0.765	1	27	46,656	764,554.86
1 cubic foot	0.0283	0.037	1	1,728	28,316.85
1 cubic inch	0.000016387	0.000021433	0.0005787	1	16.387
1 cubic centimeter	0.000001	0.000001308	0.00003531	0.0610	1

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Another example applies to foundation work. You have calculated that a concrete slab for a building requires 59 cubic yards of cement. However, the supplier works in cubic meters. How much cement would you need to order?

To convert cubic yards to cubic meters using the factor from *Table 10*, multiply the number of cubic yards (59) by the conversion factor of 0.765. The result is 45.1 cubic meters. Although slightly more cement may be ordered than necessary to eliminate any possibility of shortage, you are now working with the same volume units as the supplier.

5.3.4 Study Problems: Converting Volume Units

Convert these volumes from the Imperial system to the metric system, or vice versa. Answers should be rounded to the nearest tenth.

1. 11,600 cubic inches = _________ cubic feet
2. 1.9 cubic meters = _________ cubic centimeters
3. 512 cubic meters = _________ cubic yards
4. 7 cubic feet = _________ cubic centimeters
5. 0.2 cubic meters = _________ cubic feet

5.4.0 Temperature Units

Temperature conversions are quite often necessary. Temperature is important to many crafts, and in many different ways. However, temperature is somewhat different from other units of measure, in that most countries of the world are likely to use several temperature scales for different applications.

Temperature can be defined as the intensity level of heat. Temperature is measured in degrees on a temperature scale. In order to establish the scale, a substance is needed that always responds to reproducible conditions in the same manner. The substance used for this purpose is water. The point at which water freezes at atmospheric pressure is one reproducible condition, and the point at which water boils at standard atmospheric pressure is another.

The four temperature scales commonly used today are the Fahrenheit scale, Celsius scale, Rankine scale, and Kelvin scale (*Figure 22*). Temperature is most often measured in degrees Fahrenheit or degrees Celsius. On the Fahrenheit scale, the freezing temperature of water is 32°F and the boiling temperature is 212°F. On the Celsius scale, the freezing temperature of water is 0°C and the boiling temperature is 100°C. The temperatures at which these fixed points occur were established by the inventors of the scales. The abbreviations for each unit are shown on *Figure* 22. Each is a single capital letter.

The Rankine scale and the Kelvin scale are based on the theory that at some extremely low temperature, no molecular activity occurs. The temperature at which this condition occurs is

Around the World

Coldest Recorded Temperature

In August, 2014, physicists at Yale University succeeded in chilling molecules to the coldest temperature ever recorded. Using a process called magneto-optical trapping, they were able to reduce the temperature of strontium fluoride molecules to a temperature of −459.67°F. The final calculations indicated that the temperature achieved was less than 0.003°F above absolute zero.

The coldest natural temperature recorded at ground level is −128.6°F (−89.2°C) at a Soviet station in Antarctica in 1983.

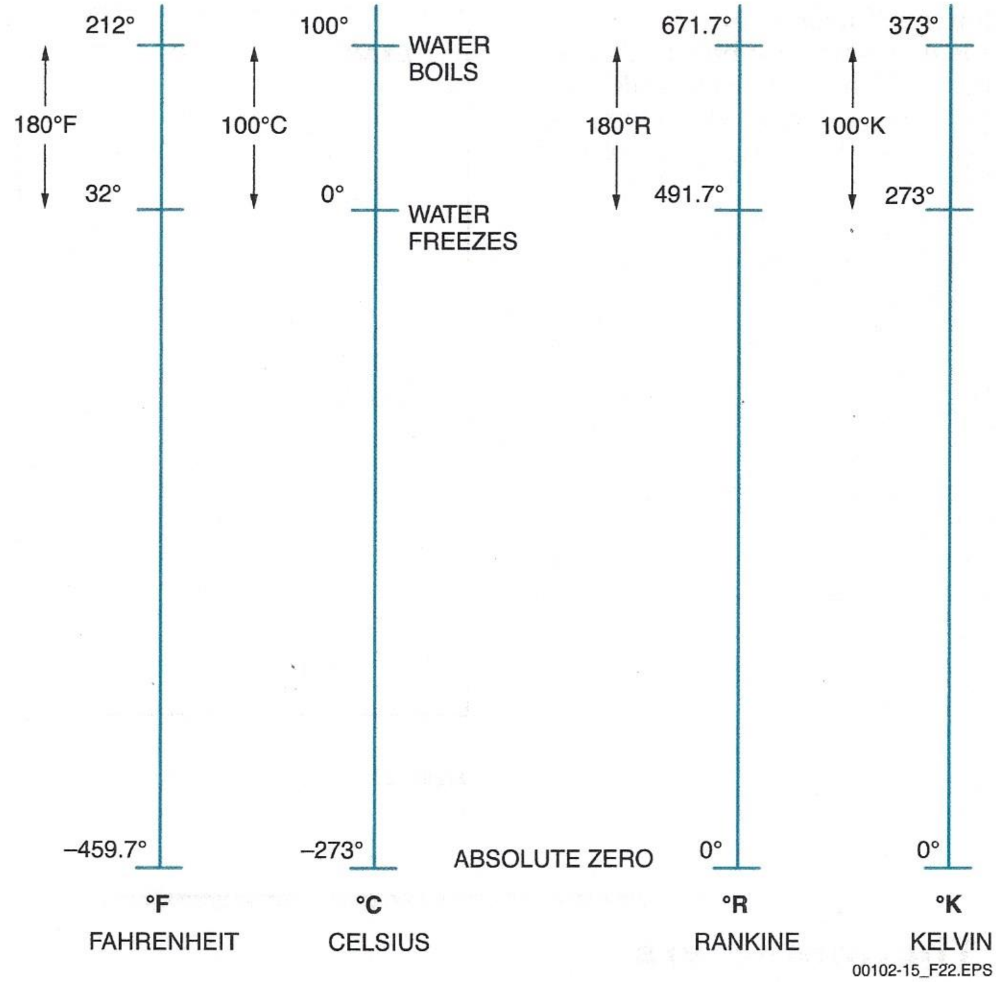

Figure 22 Comparison of temperature scales.

called absolute zero, the lowest temperature possible. Both the Rankine and Kelvin scales have their zero degree points at absolute zero. On the Rankine scale, the freezing point of water is 491.7°R and the boiling point is 671.7°R—a 180° range. This is the same number of degrees between the freezing and boiling points of water found on the Fahrenheit scale. Therefore, the increments on the Rankine scale correspond in size to the increments on the Fahrenheit scale; for this reason, the Rankine scale is sometimes called the absolute Fahrenheit scale.

On the Kelvin scale, the freezing point of water is 273°K and the boiling point is 373°K. The range between the freezing and boiling points of water is 100°. This shows the relationship between the Kelvin and Celsius scales. The increments on the Kelvin scale correspond to the increments on the Celsius scale. For this reason, the Kelvin scale is sometimes called the absolute Celsius scale. Both the Kelvin and Celsius scales are part of the metric system of measurement.

The scales of primary importance in most crafts are the Fahrenheit scale and the Celsius scale. The Rankine scale and the Kelvin scale are better suited for scientific applications.

Charts and tables are readily available from a number of sources to make the conversion of temperatures easy and fast. However, it is also beneficial to understand the math behind them. All such charts and tables are based on the mathematical relationship that follows.

On the Fahrenheit scale, there are 180 degrees between the freezing temperature and boiling temperature of water. On the Celsius scale, there are 100 degrees between the freezing and boiling temperatures of water. The relationship between the two scales can be expressed as follows:

$$\frac{\text{Fahrenheit range (freezing to boiling)}}{\text{Celsius range (freezing to boiling)}} = \frac{180°}{100°} = \frac{9}{5}$$

Therefore, one degree Fahrenheit is $\frac{5}{9}$ths of one degree Celsius and conversely, one degree Celsius is $\frac{9}{5}$ths of one degree Fahrenheit. Thus, to convert a Fahrenheit temperature to a Celsius temperature, it is necessary to subtract 32° (since 32° corresponds to 0° on the Celsius scale), and then multiply by 5/9. To convert a Celsius temperature to a Fahrenheit temperature, it is necessary to multiply by 9/5, and then add 32°C. This can be written mathematically as follows:

$$°C = \frac{5}{9}\,(°F - 32°)$$
$$°F = \left(\frac{9}{5} \times °C\right) + 32°$$

Practice these calculations to become more comfortable with using them. *Figure 23* shows two examples.

5.4.1 Study Problems: Converting Temperatures

Convert these temperatures from Fahrenheit to Celsius, or vice versa. Answers should be stated to the nearest tenth of a degree.

1. 180°F = ________°C
2. 66°F = ________°C
3. −26°C = ________°F
4. 71°C = ________°F

EXAMPLE: 77°F to °C

$$°C = \frac{5}{9}\,(77° - 32°)$$

$$°C = \frac{5}{9}\,(45°)$$

$$°C = 25°C$$

EXAMPLE: 90°C to °F

$$°F = \left(\frac{9}{5} \times 90°\right) + 32°$$

$$°F = 162° + 32°$$

$$°F = 194°F$$

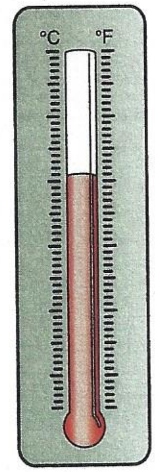

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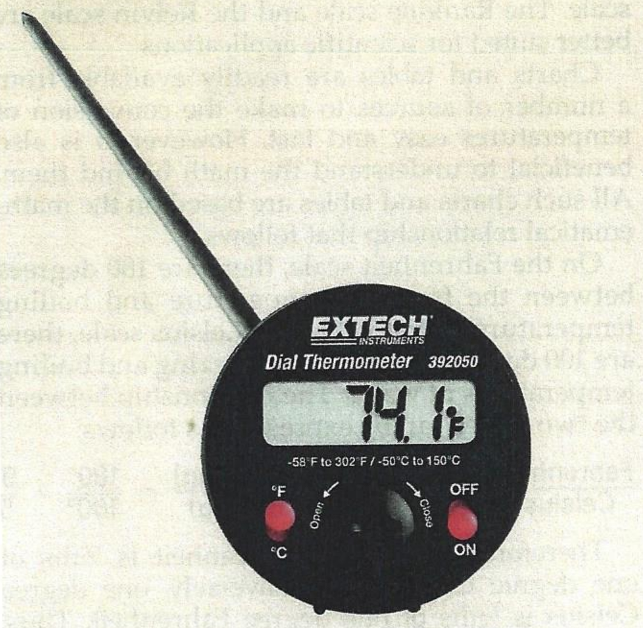
Figure 23 Sample temperature conversions.

Digital Thermometers

Thermometers with digital readouts can provide temperature readings in both Celsius and Fahrenheit. The one on the left is a temperature probe with a digital readout. The one on the right is an infrared thermometer that measures temperature based on an infrared signature from the surface. Many analog thermometers are also calibrated for both scales.



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Additional Resources

Applied Construction Math: A Novel Approach, NCCER. 2006. Upper Saddle River, NJ: Prentice Hall.

Mathematics for Carpentry and the Construction Trades, Alfred P. Webster; Kathryn B. Judy. 2001. Upper Saddle River, NJ: Prentice Hall.

Mathematics for the Trades: A Guided Approach, Robert A. Carman; Hal Saunders. 2014. Pearson Learning.

Metric-conversion.org : Metric Conversion Charts and Calculators.

5.0.0 Section Review

1. When converting from a larger metric value to a smaller one, the decimal point must move _____.
   a. to the left
   b. to the right
   c. all the way to the left end of the number
   d. all the way to the right end of the number

2. Weight is the force an object exerts on the surface of the Earth due to its mass and _____.
   a. density
   b. volume
   c. the current temperature
   d. the pull of the Earth's gravity

3. The cubic meter represents a three-dimensional object that measures _____.
   a. 10 cm × 10 cm × 10 cm
   b. 100 mm × 100 mm × 100 mm
   c. 100 cm × 100 cm × 100 cm
   d. 10 m × 10 m × 10 m

4. On the Celsius temperature scale, the difference between the freezing and boiling points of water is _____.
   a. 32 degrees
   b. 100 degrees
   c. 180 degrees
   d. 212 degrees

Section Six

6.0.0 Introduction to Geometry

Objective

Identify basic angles and geometric shapes and explain how to calculate their area and volume.

a. Identify various types of angles.
b. Identify basic geometric shapes and their characteristics.
c. Demonstrate the ability to calculate the area of two-dimensional shapes.
d. Demonstrate the ability to calculate the volume of three-dimensional shapes.

Trade Terms

Acute angle: Any angle between 0 degrees and 90 degrees.

Adjacent angles: Angles that have the same vertex and one side in common.

Angle: The shape made by two straight lines coming together at a point. The space between those two lines is measured in degrees.

Area: The surface or amount of space occupied by a two-dimensional object such as a rectangle, circle, or square.

Base: As it relates to triangles, the base is the line forming the bottom of the triangle.

Bisect: To divide into two parts that are often equal. When an angle is bisected for example, the two resulting angles are equal.

Circle: A closed curved line around a central point. A circle measures 360 degrees.

Circumference: The distance around the curved line that forms the circle.

Cube: A three-dimensional square, with the measurements in all the three dimensions being equal.

Degree: A unit of measurement for angles. For example, a right angle is 90 degrees, an acute angle is between 0 and 90 degrees, and an obtuse angle is between 90 and 180 degrees.

Diagonal: Line drawn from one corner of a rectangle or square to the farthest opposite corner.

Diameter: The length of a straight line that crosses from one side of a circle, through the center point, to a point on the opposite side. The diameter is the longest straight line you can draw inside a circle.

Equilateral triangle: A triangle that has three equal sides and three equal angles.

Formula: A mathematical process used to solve a problem. For example, the formula for finding the area of a rectangle is Side A times Side B = Area, or A × B = Area.

Isosceles triangle: A triangle that has two equal sides and two equal angles.

Obtuse angle: Any angle between 90 degrees and 180 degrees.

Opposite angles: Two angles that are formed by two straight lines crossing. They are always equal.

Perimeter: The distance around the outside of a closed shape, such as a rectangle, circle, square, or any irregular shape.

Pi: A mathematical value of approximately 3.14 (or 22⁄7) used to determine the area and circumference of circles. It is sometimes symbolized by π.

Plane geometry: The mathematical study of two-dimensional (flat) shapes.

Radius: The distance from a center point of a circle to any point on the curved line, or half the width (diameter) of a circle.

Rectangle: A four-sided shape with four 90-degree angles. Opposite sides of a rectangle are always parallel and the same length. Adjacent sides are perpendicular and are not equal in length.

Right angle: An angle that measures 90 degrees. The two lines that form a right angle are perpendicular to each other. This is the angle used most in the trades.

Right triangle: A triangle that includes one 90-degree angle.

Scalene triangle: A triangle with sides of unequal lengths.

Solid geometry: The mathematical study of three-dimensional shapes.

Square: (1) A special type of rectangle with four equal sides and four 90-degree angles. (2) The product of a number multiplied by itself. For example, 25 is the square of 5; 16 is the square of 4.

Straight angle: A 180-degree angle or flat line.

Triangle: A closed shape that has three sides and three angles.

Vertex: A point at which two or more lines or curves come together.

Geometry might sound complicated, but it is really made up of common things you are already familiar with—**circles**, **triangles**, **squares**, and **rectangles**, for example. The construction industry is based on a world of measurements and shapes. It is important to recognize basic shapes and understand them mathematically in order to make use of geometry in your chosen craft.

The first portion of this section focuses on **plane geometry**. In plane geometry, the shapes are two-dimensional. These shapes have length and width only. In **solid geometry**, also known as 3D geometry, shapes have three dimensions, including height.

6.1.0 Angles

An **angle** is an important term in the construction trades. It is used by all building trades to describe the shape made by two straight lines that meet at a point called the **vertex**. Angles are measured in **degrees** to describe the relationship between the two lines. To measure angles, a tool called a protractor is used. The following are the typical angles (*Figure 24*) you will measure in construction:

- **Acute angle** – An angle that measures between 0 and 90 degrees. The most common acute angles are 30, 45, and 60 degrees.
- **Right angle** – An angle that measures 90 degrees. The two lines that form the right angle are perpendicular to each other. Imagine the shape of a capital letter L. This is a right angle, because the sides of the L are perpendicular to one another. This is the angle used most often in the construction trades. A right angle is indicated in plans or drawings with a square symbol at the vertex, as shown in *Figure 24* .
- **Obtuse angle** – An angle that measures between 90 and 180 degrees.
- **Straight angle** – A straight angle measures precisely 180 degrees (a flat line).
- **Adjacent angles** – These angles have the same vertex and one side in common. Adjacent refers to objects that are next to each other.
- **Opposite angles** – Angles formed by two straight lines that cross are opposite. Opposite angles are always equal.

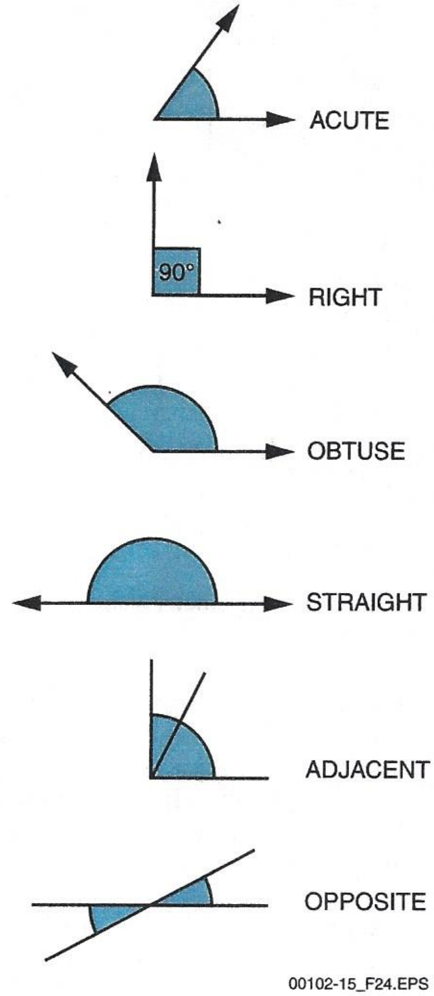


Figure 24 Types of angles.

6.2.0 Shapes

Common shapes that are essential to your work in the trades include rectangles, squares, triangles, and circles (*Figure 25*).

6.2.1 Rectangle

A rectangle is a four-sided shape with four 90-degree angles. The sum of all four angles in any rectangle is 360 degrees. A rectangle has two pairs of equal sides that are parallel to each other. The **diagonals** of a rectangle are always equal: diagonals are lines connecting opposite corners. If you cut a rectangle on the diagonal, you will have two identical **right triangles**, as shown in *Figure 26*. All right triangles have one 90-degree angle.

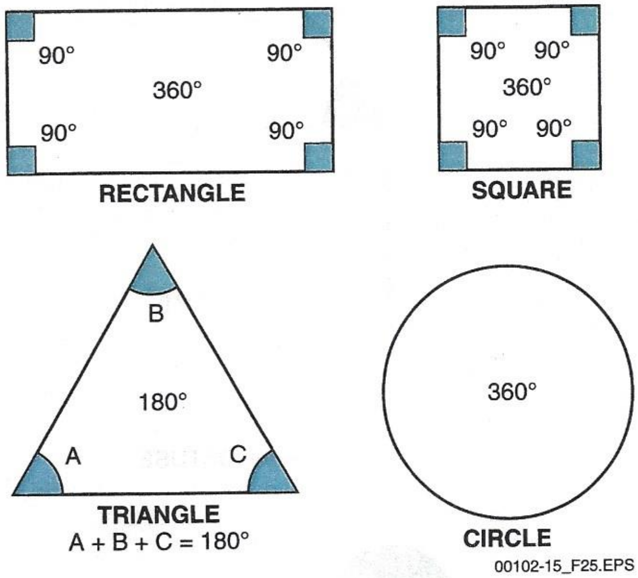


Figure 25 Common plane geometry shapes.

6.2.2 Square

A square is a type of rectangle with four sides of equal length and four 90-degree angles. The sum of all four angles in all squares is 360 degrees. If you cut a square on the diagonal between opposite corners, you will have two right triangles. Each right triangle will have two 45-degree angles and one 90-degree angle, as shown in *Figure 27*.

When measuring the outside lines of a rectangle or a square, you are determining the **perimeter**. The perimeter is the distance around any two-dimensional figure, but it also applies to some three-dimensional figures. You may need to calculate the perimeter of a shape to measure, mark, and cut the right amount of material. For example, if you need to install molding along all four walls of a room, the perimeter measurement must be known to purchase the correct amount of material. If the room is 14 feet by 12 feet, you would calculate: 14 + 12 + 14 + 12 = 52 feet of molding. Another way to calculate it would be (2 × 14 feet) + (2 × 12 feet) = 52 feet.

Since a square has four sides of equal length, any side can be measured and multiplied by four to determine the perimeter.

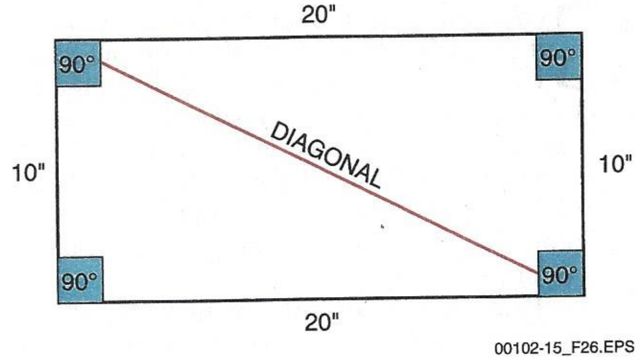


Figure 26 Cutting a rectangle on the diagonal produces two right triangles.

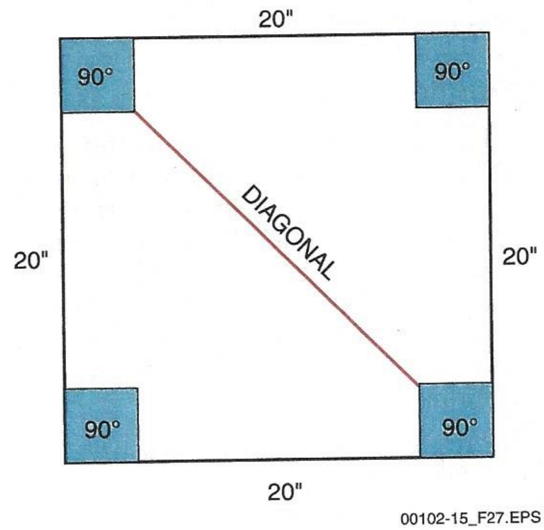


Figure 27 Cutting a square on the diagonal produces two right triangles.

Did You Know?

Rope Stretchers

The word *geometry* comes from two Greek words: *geos*, meaning "land", and *metrein*, meaning "to measure."

In ancient Egypt, most farms were located beside the Nile River. Every year during the flood season, the Nile overflowed its banks and deposited mineral-rich silt over the farmland. But the floodwaters destroyed the markers used to establish property lines between farms. When this happened, the farms had to be measured again. Men called rope stretchers re-marked the property lines each year.

The rope stretchers calculated distances and directions using ropes that had equally spaced knots tied along the length of the rope. Stretching the ropes to measure distances on level land was easy. In many cases, however, the rope stretchers had to measure property lines from one side of a hill to the opposite side or across a pond.

The hills and ponds made this measurement difficult. To adjust for the uneven land or ponds that stood in the way, rope stretchers determined new ways of measuring. Such discoveries became the foundation of geometry.

6.2.3 Triangle

A triangle is a closed shape that has three sides and three angles. Although the angles in a triangle can vary, the sum of the three angles is always 180 degrees (*Figure 28*). The following are different types of triangles you will use in construction:

- *Right triangle* – A right triangle has one 90-degree angle.
- **Equilateral triangle** – An equilateral triangle has three equal angles and three equal sides.
- **Isosceles triangle** – An isosceles triangle has two equal angles and two sides equal in length. A line that **bisects** (runs from the center of the **base** of the triangle to the highest point) an isosceles triangle creates two adjacent right angles.
- **Scalene triangle** – A scalene triangle has three sides of unequal lengths.

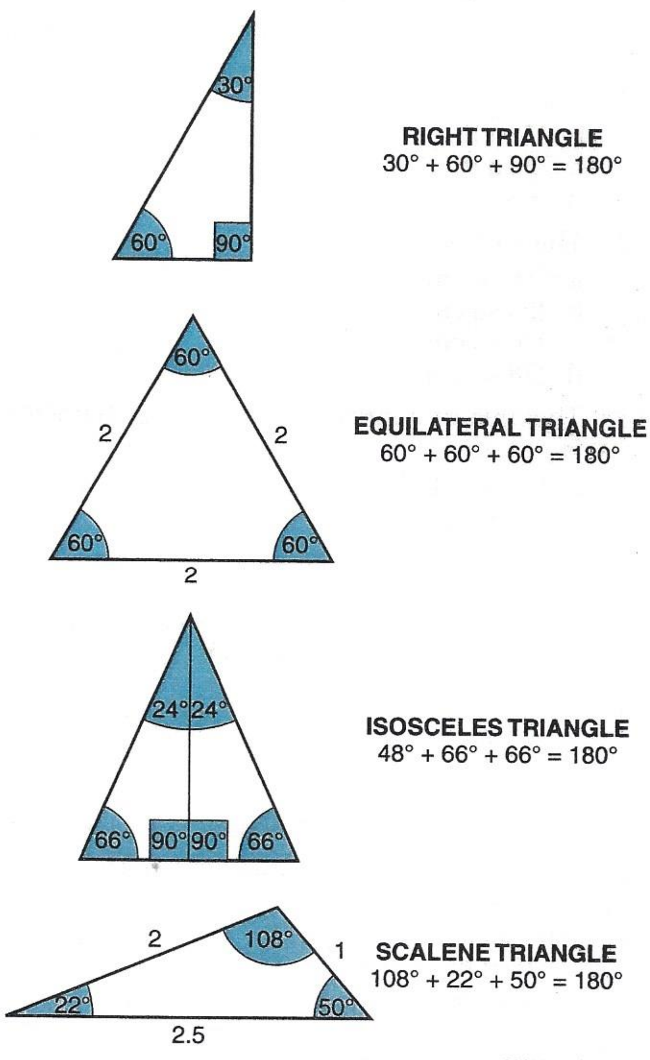


Figure 28 The sum of a triangle's three angles always equals 180 degrees.

6.2.4 Circle

A circle is a closed curved line around a center point. Every point on the curved line is exactly the same distance from the center point. A circle measures 360 degrees. The following terms apply to circles (*Figure 29*):

- **Circumference** – The circumference of a circle is the length of the closed curved line that forms the circle. The **formula** for finding circumference is **pi** (3.14) × **diameter**.
- *Diameter* – The diameter of a circle is the length of a straight line that crosses from one side of the circle through the center point to a point on the opposite side. The diameter is the longest straight line you can draw inside a circle.
- *Pi* or π – pi is a mathematical constant value of approximately 3.14 (or $22/7$) used to determine the area and circumference of circles.
- **Radius** – The radius of a circle is the length of a straight line from the center point of the circle to any point on the closed curved line that forms the circle. It is equal to half the diameter.

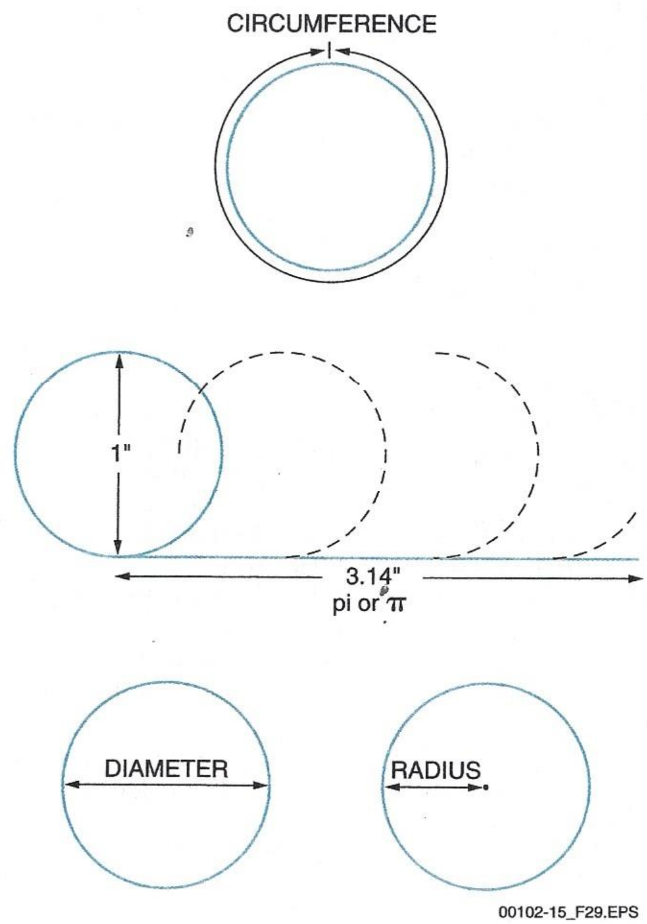


Figure 29 Measurements that apply to circles.

6.3.0 Calculating the Area of Shapes

Area is the measurement of the surface of a two-dimensional object. For example, you must calculate the area of a shape, such as a floor or a wall, to order the proper amount of material, such as carpeting or paint. Squared units of measure describe the amount of surface area. Area measurements in the Imperial system are typically in square inches (sq in or in^2), square feet (sq ft or ft^2), and square yards (sq yd or yd^2). Area measurements in the metric system are often in square centimeters (sq cm or cm^2) and square meters (sq m or m^2). When larger areas are involved, such as land, the units may be in square miles or square kilometers.

- 1 square inch = 1 inch × 1 inch = 1 $inch^2$
- 1 square foot = 1 foot × 1 foot = 1 $foot^2$
- 1 square yard = 1 yard × 1 yard = 1 $yard^2$
- 1 square centimeter = 1 cm × 1 cm = 1 cm^2
- 1 square meter = 1 m × 1 m = 1 m^2

You must be able to calculate the area of basic shapes. Mathematical formulas make this very easy to do. In the *Appendix*, you will find formulas for calculating the areas of various shapes. You need to become familiar with these formulas at this stage in your training. The formulas for calculating the most common shapes are presented here:

- *The area of a rectangle* = length × width. For example, you have to paint a wall that is 20 feet long and 8 feet high. To calculate the area, multiply 20 ft × 8 ft = 160 sq ft.
- *The area of a square* = length × width. However, remember that all sides of a square are equal. For example, you have to tile a 12-meter square room. The area is 12m × 12m = 144 sq m.
- *The area of a circle* = pi × $radius^2$. In this formula, use the mathematical constant pi, which has an approximate value of 3.14. Multiply pi by the radius of the circle squared. For example, to find the area of a circular driveway to be sealed, you must first find the radius. If the radius is 20 feet, the calculation is 3.14 × $(20 \text{ ft})^2$ or 3.14 × 400 sq ft = 1,256 sq ft.
- *The area of a triangle* = ½ × base × height. The base is the side the triangle sits on. The height is the length of the triangle from its base to the highest point. For example, you have to install a piece of siding on a triangular section of a building. You find the triangle has a base of 2 feet and a height of 4 feet. The calculation is ½ × 2 ft × 4 ft = 4 sq ft.

Diagonals

Diagonals have a number of uses. If you have to make sure that a surface is a true rectangle with 90-degree corners, you can measure the diagonals to find out. For example, before applying a piece of sheathing, you must make sure it is a true rectangle. Using your tape measure, find the length of the sheathing from one corner to the opposite corner. Now, find the length of the other two opposing corners. Do the diagonals match? If so, the piece of sheathing is a true rectangle. If not, the piece is not a true rectangle and will cause problems when you install it.

6.3.1 Study Problems: Calculating Area

1. The area of a rectangle that is 8 feet long and 4 feet wide is _____.
   a. 12 sq ft
   b. 22 sq ft
   c. 32 sq ft
   d. 36 sq ft
2. The area of a 16 cm square is _____.
   a. 256 sq cm
   b. 265 sq cm
   c. 276 sq cm
   d. 278 sq cm
3. The area of a circle with a 14-foot diameter is _____.
   a. 15.44 sq ft
   b. 43.96 sq ft
   c. 153.86 sq ft
   d. 196 sq ft
4. The area of a triangle with a base of 4 centimeters and a height of 6 cm is _____.
   a. 12 sq cm
   b. 24 sq cm
   c. 32 sq cm
   d. 36 sq cm
5. The area of a rectangle that is 14 meters long and 5 meters wide is _____.
   a. 60 sq m
   b. 65 sq m
   c. 70 sq m
   d. 75 sq m

6.4.0 Volume of Three-Dimensional Shapes

Volume is the amount of space occupied in three dimensions. To calculate volume, you must use three measurements: length, width, and height. One dimension, usually height, may be referred to as depth or thickness. Cubic units of measure describe the volume of different spaces. Measurements in the Imperial system are in cubic inches (cu in or in^3), cubic feet (cu ft or ft^3), and cubic yards (cu yd or yd^3). Metric measurements include cubic centimeters (cu cm or cm^3) and cubic meters (cu m or m^3).

- 1 cubic inch = 1 inch × 1 inch × 1 inch = 1 inch^3
- 1 cubic foot = 1 foot × 1 foot × 1 foot = 1 foot^3
- 1 cubic yard = 1 yard × 1 yard × 1 yard = 1 yard^3
- 1 cubic centimeter = 1 centimeter × 1 centimeter × 1 centimeter = 1 cm^3
- 1 cubic meter = 1 meter × 1 meter × 1 meter = 1 m^3

One very important fact to keep in mind is that all three dimensions used in calculating volume must be in the same units. For example, a concrete slab may have dimensions of 4 feet × 6 feet × 6 inches. If these three values are multiplied together to calculate volume, there will be a major error in the result. The calculation would result in 144 ft^3. The final dimension in inches must be converted to feet first, or the other two must be converted to inches. In this case, 6 inches is equal to 0.5 feet. The correct result then would be a volume of 12 ft^3. As you can see, the first answer is incorrect by a factor of 12 due to mismatched units.

Note that there are many opinions about the proper name for a rectangle in its three-dimensional form. The concrete slab just discussed would be such a figure. In a geometry class, you may hear it referred to as a rectangular parallel pipe, rectangular prism, or a three-dimensional orthotope. To avoid confusion over such rarely-heard terms, we will simply refer to them here as three-dimensional rectangles.

You must be able to calculate the volume of common shapes in many different crafts. The following sections provide the mathematical formulas that make this easy to do. In the *Appendix*, you will find a list of the formulas for calculating the volumes of a number of shapes. It is best to become familiar with these formulas at this stage in your training. Remember to ensure that all dimensions are in matching units before multiplying.

6.4.1 Three-Dimensional Rectangles

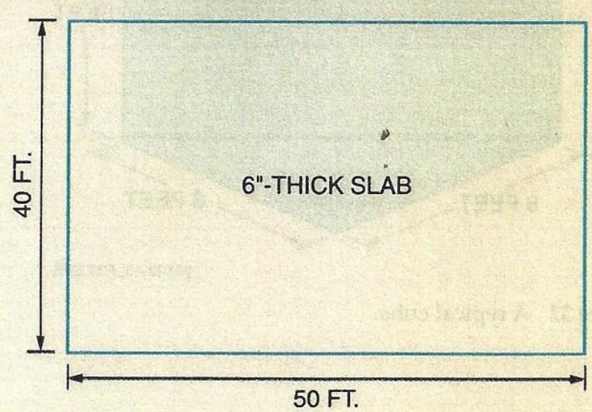
The volume of a three-dimensional rectangle is calculated by multiplying length × width × depth. For example, you might need to order the right amount of concrete (which is delivered by the cubic yard) for a slab that is 20 feet long and 8 feet wide and 4 inches thick (*Figure 30*). You must know the total volume of the slab. To calculate this, perform the following steps:

Geometry Practical Application

Before a metal building can be built, you must first pour a slab and install anchor bolts. The anchor bolts will be used to attach the structure to the slab. Because cement comes in cubic yards (yd^3), a volume measure, you must determine how much cement you need to pour the slab. To determine volume, multiply the length times the width times the depth, or thickness, of the slab. If the area of the slab is already known, simply multiply the area times the depth.

Solution

Volume = area × depth
Area of slab = 50' × 40' = 2,000 square feet (ft^2)
Slab depth = 6"
(convert inches to feet:
6" = 6/12 = ½ foot)
Volume = 2,000 ft^2 × ½ foot = 1,000 ft^3
Volume of slab = 1,000 ft^3



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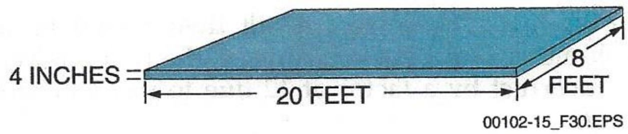


Figure 30 Volume of a proposed concrete slab.

Step 1 Convert inches to feet.

20 ft × 8 ft × (4 in ÷ 12) =

Step 2 Multiply length × width × depth.

20 ft × 8 ft × 0.33 ft = 52.8 cu ft

Step 3 Convert cubic feet to cubic yards.

52.8 cu ft ÷ 27 (cu ft per cu yd) = 1.96 cu yd of concrete

6.4.2 Cubes

A **cube** is a three-dimensional square. The volume of a cube is basically calculated in the same way as three-dimensional rectangles: length × width × depth. However, all sides of a cube are equal in length (*Figure 31*). To find the volume of a cube, you can cube (multiply the number by itself three times) one of the dimensions. Perform the following steps to determine the volume of a cube:

Step 1 Determine the volume of an 8-foot cube.

8 ft × 8 ft × 8 ft = 512 cu ft

Step 2 If necessary for the task, convert cubic feet to cubic yards.

512 cu ft ÷ 27 = 18.96 cu yd of concrete

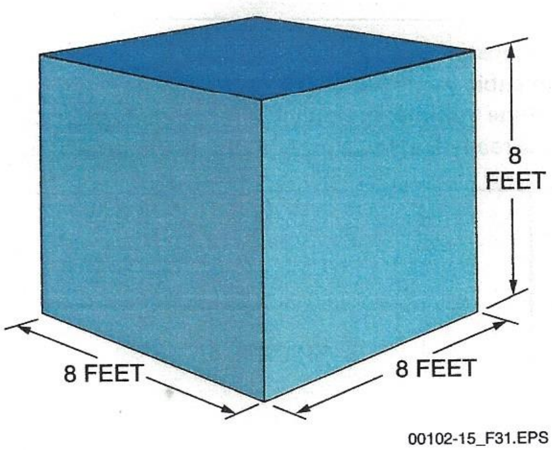


Figure 31 A typical cube.

Around the World

Pi

Pi is a mathematical constant that describes the relationship between a circle's circumference and its diameter. It is usually used in its abbreviated form, using only two to four digits to the right of the decimal point. However, it is a never-ending number, and the digits to the right of the decimal never develop into a repetitive pattern. For many years, ever-increasing records have been set in memorizing the digits. In 2005, a gentleman by the name of Lu Chao in China accurately recited 67,890 digits past the decimal point in 24 hours, 4 minutes without any aids.

6.4.3 Cylinders

The volume of a cylinder is calculated using the following formula: pi × radius^2 × height. You may also see this written as area = $\pi r^2 h$, where h represents the height. This is the same as the formula for finding the area of a circle with the added dimension of height. For example, you must find the volume of a cylinder that is 22 feet in diameter and 10 feet high (*Figure 32*):

Step 1 First, calculate the area of the circle using πr^2. Since the diameter is 22 feet, the radius will be 11 feet.

Area of the circle = 3.14 × 11^2 = 379.94 sq ft

Step 2 Then calculate the volume (area × height).

379.94 sq ft × 10 ft = 3,799.4 cu ft

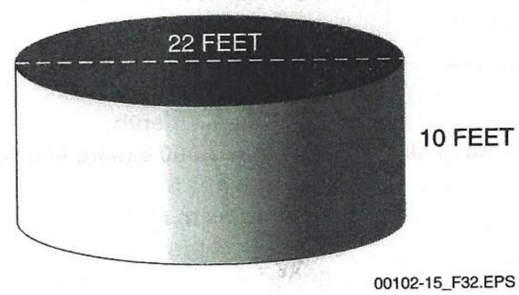


Figure 32 A cylinder.

6.4.4 Triangular Prisms

The volume of a triangular prism is calculated using the following formula: 0.5 × base × height × depth (thickness). Note that this is not a pyramid, but a triangle that has depth and is consistent in dimension, such as the one shown in *Figure 33*. In this example, you must fill a triangular shape that has a base of 6 cm, a height of 12 cm, and a depth of 2 cm:

Step 1 Calculate the area of the flat triangle first:

0.5 × 6 × 12 = 36 sq cm area

Step 2 Then calculate the volume of the prism, by adding the factor of depth:

36 sq cm × 2 cm = 72 cu cm

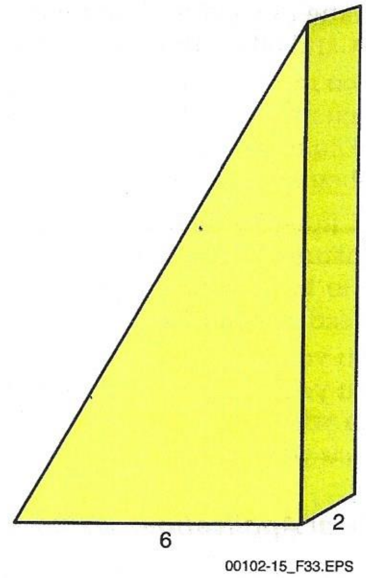


Figure 33 Volume of a triangular prism.

6.4.5 Study Problems: Calculating Volume

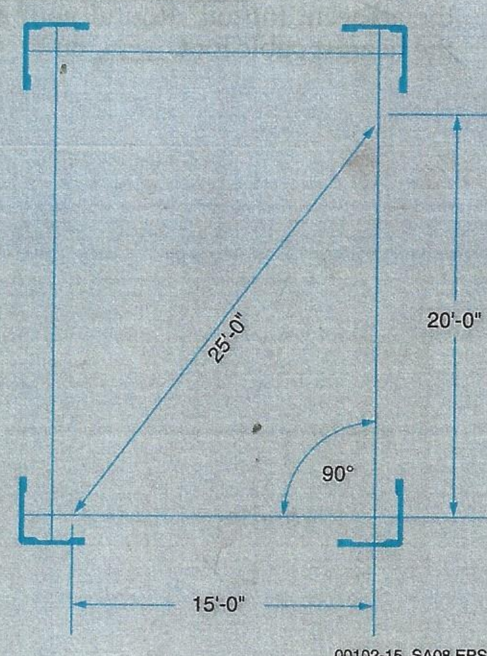
1. The volume of a rectangular shape 5 feet high, 6 feet thick, and 13 feet long is _____.
   a. 24 cu ft
   b. 43 ft
   c. 95 ft
   d. 390 cu ft
2. The volume of a 3 cm cube is _____.
   a. 6 cu cm
   b. 9 cu cm
   c. 12 cu cm
   d. 27 cu cm
3. The volume of a triangular prism that has a 6-inch base, a 2-inch height, and a 4-inch depth is _____.
   a. 12 sq in
   b. 24 cu in
   c. 36 cu in
   d. 48 sq in

Did You Know?

3-4-5 Rule

The 3-4-5 rule is based on the Pythagorean theorem, and it has been used in building construction for centuries. This simple method for laying out or checking right angles requires only the use of a tape measure. The numbers 3-4-5 represent dimensions in feet that describe the sides of a right triangle. Right triangles that are multiples of the 3-4-5 triangle are commonly used, such as 9-12-15, 12-16-20, and 15-20-25. The specific multiple used is determined by the relative distances involved in the job being laid out or checked.

Refer to the figure for an example of the 3-4-5 theory using the multiples of 15-20-25. In order to square, or check, a corner, first measure and mark 15'-0" down the line in one direction, then measure and mark 20'-0" down the line in the other direction. The distance measured between the 15'-0" and 20'-0" points must be exactly 25'-0" to ensure that the angle is a perfect right (90 degree) angle.



4. The volume of a cylinder that is 6 meters in diameter and 60 centimeters high is _____.
   a. 16.96 cu m
   b. 18.23 cu m
   c. 1130.4 cu m
   d. 6782.4 cu m
5. If gravel must be distributed across an area that measures 17 feet square and the gravel layer is to be 6 inches thick, the volume of gravel needed would be _____.
   a. 3.77 cu yds
   b. 5.35 cu yds
   c. 102 cu yds
   d. 144.5 cu yds

6.4.6 Practical Applications Using Volume

The following examples illustrate some of the practical applications a tradesperson may encounter on the job site that requires an understanding of geometry-based principals to solve. Use the information provided to solve each problem. Be sure to show all of your work.

1. Use *Figure 34* to calculate the volume of a section of round sheet metal pipe. The volume of the section shown = _____ cubic inches.
2. To pour the concrete sidewalk shown in *Figure 35,* approximately how many cubic feet of topsoil will you need to remove for the 4"-thick sidewalk if the owner wants the finish surface of the sidewalk to be level with the adjacent topsoil? Round your answer to the nearest cubic foot. _____ cu ft

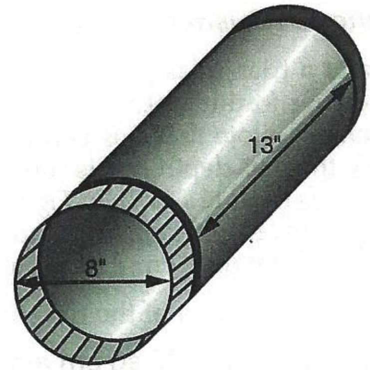


Figure 34 Finding the volume of a pipe section.

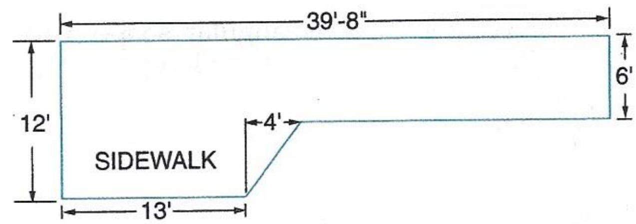


Figure 35 Removing topsoil for a sidewalk.

Additional Resources

Applied Construction Math: A Novel Approach, NCCER. 2006. Upper Saddle River, NJ: Prentice Hall.

Mathematics for Carpentry and the Construction Trades, Alfred P. Webster; Kathryn B. Judy. 2001. Upper Saddle River, NJ: Prentice Hall.

Mathematics for the Trades: A Guided Approach, Robert A. Carman; Hal Saunders. 2014. Pearson Learning.

Metric-conversion.org : Metric Conversion Charts and Calculators.

6.0.0 Section Review

1. An angle of 66 degrees is considered a(n) _____.
   a. right angle
   b. straight angle
   c. obtuse angle
   d. acute angle

2. What is the difference between circumference and perimeter?
   a. The circumference relates only to shapes with angles, while the perimeter relates only to circles.
   b. The perimeter relates to all shapes, while the circumference relates only to circles.
   c. The perimeter represents one-half of the circumference.
   d. The circumference is never larger than the perimeter.

3. Calculate the area of a rectangle that is 27.3 meters × 9.3 meters.
   a. 245.7 m^3
   b. 251.1 m^2
   c. 253.89 m^2
   d. 253.89 m^3

4. Calculate the volume of a shipping container that is 26' long × 13'8" wide × 3' deep.
   a. 355.32 ft^3
   b. 355.32 ft^2
   c. 1,065.48 ft^3
   d. 1,065.48 ft^2

SUMMARY

Mathematics is not just something you need to learn to survive your days in school. The construction environment requires math every day to get a project done. Whether you are cutting stock, charging an air conditioning unit with refrigerant, or installing electrical systems, you will need math skills on the job. Basic operations such as addition, subtraction, multiplication, and division are the keys to completing these tasks. However, more complex mathematical operations will be necessary for a number of tasks, such as planning a piping offset. Being competent and comfortable with math increases your value to an employer, which helps ensure your job security.

Review Questions

1. The number matching the words "two thousand, six hundred eighty-nine" is _____.
   a. 2,286
   b. 2,689
   c. 6,289
   d. 20,689

Solve Questions 2 through 5 without using a calculator.

2. A bricklayer lays 649 bricks the first day, 632 the second day, and 478 the third day. During the three-day period, the bricklayer laid a total of _____.
   a. 1,759
   b. 1,760
   c. 1,769
   d. 1,770

3. A total of 1,478 feet of cable was supplied for a job. Only 489 feet were installed. How many feet of cable remain?
   a. 978
   b. 980
   c. 989
   d. 1,099

4. A worker has been asked to deliver 15 scaffolds to each of 26 different sites. The worker will deliver a total of _____.
   a. 120
   b. 240
   c. 375
   d. 390

5. Your company has 400 rolls of insulation that must be equally distributed to 5 different job sites. How many rolls of insulation will need to go to each site?
   a. 8
   b. 20
   c. 80
   d. 208

6. Which of the following fractions is an equivalent fraction for $\frac{3}{8}$?
   a. $\frac{3}{64}$
   b. $\frac{6}{64}$
   c. $\frac{24}{64}$
   d. $\frac{36}{64}$

7. The lowest common denominator for the fractions $\frac{5}{64}$ and $\frac{8}{32}$ is _____.
   a. 8
   b. 16
   c. 24
   d. 32

Perform the calculations for Questions 8 and 9. Reduce the answers to their lowest terms.

8. $\frac{3}{8} + \frac{9}{16} =$ _____
   a. $\frac{3}{4}$
   b. $\frac{12}{16}$
   c. $\frac{7}{8}$
   d. $\frac{15}{16}$

9. $\frac{11}{32} - \frac{2}{8} =$ _____
   a. $\frac{3}{32}$
   b. $\frac{1}{8}$
   c. $\frac{9}{32}$
   d. $\frac{19}{32}$

10. Put the following decimals in order from smallest to largest: 0.402, 0.420, 0.042, 0.442
    a. 0.420, 0.042, 0.442, 0.402
    b. 0.042, 0.402, 0.420, 0.442
    c. 0.442, 0.420, 0.402, 0.042
    d. 0.042, 0.402, 0.442, 0.420

11. Two coatings have been applied to a pipe. The first coating is 51.5 nanometers thick; the second coating is 89.7 nanometers thick. How thick is the combined coating?
    a. 141.2 nanometers
    b. 142.12 nanometers
    c. 144.2 nanometers
    d. 145.02 nanometers

12. $3.53 \times 9.75 =$ _____
    a. 12.28
    b. 34.42
    c. 36.14
    d. 48.13

13. It costs $2.37 to paint one square foot of wall. You need to paint a wall that measures 864.5 square feet. To paint that wall it will cost _____. (Round your answer to the nearest hundredth.)

a. $204.88
b. $2,048.87
c. $2,848.88
d. $2,888.86

14. 89.435 ÷ 0.05 = _____

a. 1788.7
b. 17.887
c. 4.47175
d. 447.175

Solve Questions 15 and 16 without using a calculator.

15. 13.9% = _____

a. 0.009
b. 0.013
c. 0.139
d. 1.39

16. Convert 14.75 to its equivalent fraction expressed in lowest terms.

a. $\frac{1475}{100}$
b. $\frac{295}{20}$
c. $\frac{59}{4}$
d. $\frac{147}{4}$

17. The arrow in Review Question *Figure 1* is pointing to _____.

a. $4\frac{3}{8}$ inches
b. $4\frac{1}{4}$ inches
c. $4\frac{1}{2}$ inches
d. $4\frac{5}{8}$ inches

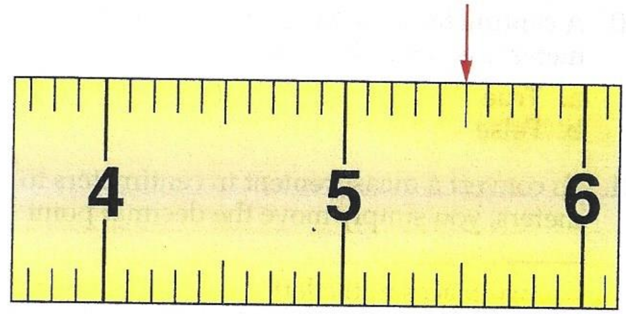


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Figure 2

18. The arrow in Review Question *Figure 2* is pointing to _____.

a. 5.2 cm
b. 5.3 cm
c. 5.4 cm
d. 5.5 cm

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Figure 3

19. The red background around the number 32 in Review Question *Figure 3* indicates _____.

a. that the tape measure is metric
b. the Imperial measurement equivalent to one meter
c. the marking for studs placed on 16-inch centers in a wall structure
d. the marking for studs placed on 24-inch centers

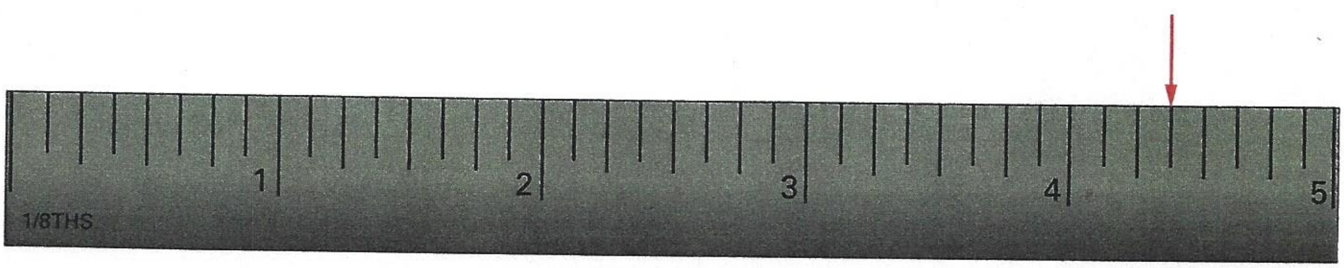

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Figure 1

20. A centimeter is 1⁄100th of a meter, while a kilometer is equal to 1000 meters.
   a. True
   b. False

21. To convert a measurement in centimeters to meters, you simply move the decimal point _____.
   a. two places to the left
   b. three places to the right
   c. three places to the left
   d. four places to the right

22. Convert 67 inches to centimeters.
   a. 17.01 cm
   b. 26.38 cm
   c. 170.18 cm
   d. 263.80 cm

23. A pound of water weighs more than a kilogram of water.
   a. True
   b. False

24. Convert the weight of 49 ounces to grams.
   a. 1.73 grams
   b. 3.06 grams
   c. 747.42 grams
   d. 1389.15 grams

25. Convert 15°F to Celsius.
   a. −9.4°C
   b. 9.4°C
   c. −59°C
   d. 59°C

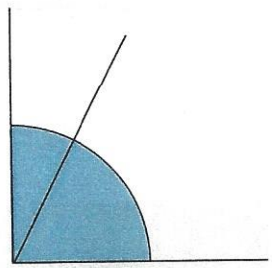

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Figure 4

26. The two angles shown in Review Question *Figure 4* are both _____.
   a. obtuse and adjacent
   b. obtuse and opposite
   c. right and opposite
   d. acute and adjacent

27. The sum of the four angles in a rectangle is always 180 degrees.
   a. True
   b. False

28. The mathematical constant pi is needed when calculating the area of a _____.
   a. circle
   b. square
   c. rectangle
   d. cube

Use a calculator to answer Questions 29 and 30.

29. The volume of a cylindrical tank that is 23 feet high with a radius of 6.25 feet is _____. (Round your answer to the nearest cubic foot)
   a. 144 cu ft
   b. 898 cu ft
   c. 903 cu ft
   d. 2,821 cu ft

30. The volume of a three-dimensional triangular prism that has a 7 cm base, a 4 cm height, and a 30 mm depth is _____. (Remember to convert all measurements to the same unit before applying the formula.)
   a. 42 cm^3
   b. 42 mm^3
   c. 82 mm^3
   d. 82 cm^3

Trade Terms Quiz

Fill in the blank with the correct term that you learned from your study of this module.

1. The ________ is equal to half the diameter of a circle.
2. A(n) ________ measures between 0 and 90 degrees.
3. When you ________ an angle, you divide it into two equal parts.
4. ________ can be measured in ft^2, in^2, cm^2, as well as other square units.
5. A(n) ________ is the shape made by two straight lines coming together at a point.
6. A(n) ________ is a curved line drawn at a consistent distance around a central point.
7. The ________ is a unit of measurement for angles.
8. The mathematical study of two-dimensional shapes is known as ________.
9. A wall that cannot be moved because it is carrying the weight of the roof is considered a ________ wall.
10. In the fraction ¾, 3 is called the ________.
11. In the same fraction ¾, 4 is called the ________.
12. A line drawn from one corner of a rectangle to the opposite corner is called a(n) ________.
13. The ________ is the longest straight line you can draw inside a circle, representing the distance across it and passing through the center point.
14. The vertical support element inside a wall to which wall finish material is attached is called a ________.
15. In the problem $17 - 8 = 9$, the number 9 is the ________.
16. Calculating the volume of three-dimensional objects, such as a cube, relies on the math associated with ________.
17. The numerical symbols from 0 to 9 are called ________.
18. In the number 7,890,342, the ________ of the 3 is three hundred.
19. Fractions having different numerators and denominators, yet they represent the same value, such as ¼ and 2⁄8, are called ________.
20. The ________ for finding the area of a triangle is ½ × base × height.
21. ¼ and ⅛ are examples of ________.
22. The point at which two or more lines come together to form an angle is called the ________.
23. 9⁄5 and 5⁄4 are examples of ________.
24. To solve the problem ¾ ÷ ½, you must ________ the second fraction and multiply.
25. In the problem $25 + 25 = 50$, the number 50 is the ________.
26. In the problem $86 \div 4$, the answer is 21 with a(n) ________ of 2.
27. A(n) ________ is one that measures between 90 and 180 degrees.
28. A(n) ________ is a combination of a whole number with a fraction or decimal.
29. ________ are angles that have the same vertex and one side in common.
30. The line that forms the bottom of a triangle is called the ________.
31. To determine the ________, measure the distance around the outside of a closed shape such as a square.
32. Complete numerical units without fractions or decimals are called ________.
33. Numbers that are less than zero are called ________.
34. Numbers that are greater than zero are called ________.
35. The type of triangle that has sides of unequal lengths is a(n) ________.
36. If a triangle has one 90-degree angle, it is a(n) ________ regardless of the other angles.

37. The shape that has three equal sides and three equal angles is called a(n) ________.

38. A(n) ________ has two equal sides and two equal angles.

39. A(n) ________ measures exactly 90 degrees.

40. A(n) ________ measures 180 degrees (basically a flat line).

41. ________ is a mathematical constant that equals approximately 3.14 or $^{22}/_{7}$.

42. A(n) ________ is a geometric figure with three sides and three angles.

43. To calculate the ________ of a cube, multiply the length times the width times the height.

44. The answer to a multiplication problem is called the ________.

45. In a division problem the answer is called the ________.

46. Any definite standard of measurement, such as a centimeter or a foot, is called a(n) ________.

47. The distance around the outside of a circle is called the ________.

48. A specific type of rectangle, with all four sides equal in length, is a(n) ________.

49. When a square is given a third dimension, equal to the other two, it becomes a(n) ________.

50. In a division problem, the number being divided is called the ________.

51. The number divided into the other number in a division problem is the ________.

52. A(n) ________ is a two-dimensional shape that has four sides, with both pairs of opposite sides being equal in length.

53. ________ can be defined as a push or pull on a surface.

54. When a number is written using a whole number plus a portion of one, with the portion separated from the whole number by a dot, it is called a(n) ________.

55. The term ________ represents the quantity of a given material that is present.

56. Two equal angles that are formed when two lines cross over each other are referred to as ________.

57. Pieces of lumber placed horizontally to support a floor or ceiling are called ________.

58. A mathematical statement that indicates that two different expressions are equal, separating them with an equal sign, is called a(n) ________.

Trade Terms

Acute angle
Adjacent angles
Angle
Area
Base
Bisect
Circle
Circumference
Cube
Decimal
Degree
Denominator
Diagonal
Diameter
Difference
Digit
Dividend
Divisor
Equation
Equilateral triangle
Equivalent fractions
Force
Formula
Fraction
Improper fraction
Invert
Isosceles triangle
Joists
Loadbearing
Mass
Mixed number
Negative numbers
Numerator
Obtuse angle
Opposite angles
Perimeter
Pi
Place value
Plane geometry
Positive numbers
Product
Quotient
Radius
Rectangle
Remainder
Right angle
Right triangle
Scalene triangle
Solid geometry
Square
Straight angle
Stud
Sum
Triangle
Unit
Vertex
Volume
Whole numbers

MULTIPLICATION TABLE

Trace across and down from the numbers that you want to multiply, and find the answer. In the example, $7 \times 7 = 49$.

	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

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Appendix B

Conversion Factors and Common Formulas

COMMON MEASURES
WEIGHT UNITS
1 ton = 2,000 pounds
1 pound = 16 dry ounces
LENGTH UNITS
1 yard = 3 feet
1 foot = 12 inches
VOLUMES
1 cubic yard = 27 cubic feet
1 cubic foot = 1,728 cubic inches
1 gallon = 4 quarts
1 quart = 2 pints
1 pint = 2 cups
1 cup = 8 fluid ounces
AREA UNIT
1 square yard = 9 square feet
1 square foot = 144 square inches

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PREFIX	SYMBOL	NUMBER	MULTIPLICATION FACTOR
giga	G	billion	$1{,}000{,}000{,}000 = 10^{9}$
mega	M	million	$1{,}000{,}000 = 10^{6}$
kilo	k	thousand	$1{,}000 = 10^{3}$
hecto	h	hundred	$100 = 10^{2}$
deka	da	ten	$10 = 10^{1}$
			BASE UNITS $1 = 10^{0}$
deci	d	tenth	$0.1 = 10^{-1}$
centi	c	hundredth	$0.01 = 10^{-2}$
milli	m	thousandth	$0.001 = 10^{-3}$
micro	μ	millionth	$0.000001 = 10^{-6}$
nano	n	billionth	$0.000000001 = 10^{-9}$

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WEIGHT UNITS		
1 kilogram	=	1,000 grams
1 hectogram	=	100 grams
1 dekagram	=	10 grams
1 gram	=	1 gram
1 decigram	=	0.1 gram
1 centigram	=	0.01 gram
1 milligram	=	0.001 gram
LENGTH UNITS		
1 kilometer	=	1,000 meters
1 hectometer	=	100 meters
1 dekameter	=	10 meters
1 meter	=	1 meter
1 decimeter	=	0.1 meter
1 centimeter	=	0.01 meter
1 millimeter	=	0.001 meter
VOLUME UNITS		
1 kiloliter	=	1,000 liters
1 hectoliter	=	100 liters
1 dekaliter	=	10 liters
1 liter	=	1 liter
1 deciliter	=	0.1 liter
1 centiliter	=	0.01 liter
1 milliliter	=	0.001 liter

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US TO METRIC CONVERSIONS		
WEIGHTS		
1 ounce	=	28.35 grams
1 pound	=	435.6 grams or 0.4536 kilograms
1 (short) ton	=	907.2 kilograms
LENGTHS		
1 inch	=	2.540 centimeters
1 foot	=	30.48 centimeters
1 yard	=	91.44 centimeters or 0.9144 meters
1 mile	=	1.609 kilometers
AREAS		
1 square inch	=	6.452 square centimeters
1 square foot	=	929.0 square centimeters or 0.0929 square meters
1 square yard	=	0.8361 square meters
VOLUMES		
1 cubic inch	=	16.39 cubic centimeters
1 cubic foot	=	0.02832 cubic meter
1 cubic yard	=	0.7646 cubic meter
LIQUID MEASUREMENTS		
1 (fluid) ounce	=	0.095 liter or 28.35 grams
1 pint	=	473.2 cubic centimeters
1 quart	=	0.9263 liter
1 (US) gallon	=	3,785 cubic centimeters or 3.785 liters
TEMPERATURE MEASUREMENTS		
To convert degrees Fahrenheit to degrees Celsius, use the following formula: C = 5/9 × (F – 32).		

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METRIC TO US CONVERSIONS		
WEIGHTS		
1 gram (G)	=	0.03527 ounces
1 kilogram (kg)	=	2.205 pounds
1 metric ton	=	2,205 pounds
LENGTHS		
1 millimeter (mm)	=	0.03937 inches
1 centimeter (cm)	=	0.3937 inches
1 meter (m)	=	3.281 feet or 1.0937 yards
1 kilometer (km)	=	0.6214 miles
AREAS		
1 square millimeter	=	0.00155 square inches
1 square centimeter	=	0.155 square inches
1 square meter	=	10.76 square feet or 1.196 square yards
VOLUMES		
1 cubic centimeter	=	0.06102 cubic inches
1 cubic meter	=	35.31 cubic feet or 1.308 cubic yards
LIQUID MEASUREMENTS		
1 cubic centimeter (cm^3)	=	0.06102 cubic inches
1 liter (1,000 cm^3)	=	1.057 quarts, 2.113 pints, or 61.02 cubic inches
TEMPERATURE MEASUREMENTS		
To convert degrees Celsius to degrees Fahrenheit, use the following formula: F = (9/5 × C) + 32.		

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Common Formulas

PRESSURE:

Absolute pressure = gauge pressure + atmospheric pressure. The atmospheric pressure at sea level is typically accepted as 14.7 psi.

Gauge pressure to static pressure: $P = hd/144$, where P = the pressure in psi; h = the height of the column in feet; d = the density of the liquid in pounds per cubic foot.

CIRCLE:

Area = πr^2, where r is the radius

Circumference = πd, where d is the diameter

SQUARES/RECTANGLES:

Area = length × width

Volume = length × width × height

TEMPERATURE CONVERSION:

$°C = 5/9\ (°F - 32°)$

$°F = (9/5 \times °C) + 32°$

SEQUENCE OF OPERATIONS:

PEMDAS = parenthesis, exponents, multiplication, division, addition, and subtraction

AIR FLOW VOLUME CHANGE:

New cfm = new rpm × existing cfm/existing rpm

TRIANGLES:

Area = $(ab)/2$, where a is the base length and b is the height

Pythagorean theorem for right triangles:

$c^2 = a^2 + b^2$, or $c = \sqrt{a^2 + b^2}$, where c is the hypotenuse of the triangle. The hypotenuse is the side opposite the right angle. Sides a and b are adjacent to the angle.

PYRAMID:

Volume = $(Ah)/3$, where A is the area of the base and h is the height

CYLINDER:

Volume = $\pi r^2 h$, where r is the radius of the base and h is the height

CONE:

Volume = $(\pi r^2 h)/3$, where r is the radius of the base and h is the height

SPHERE:

Volume = $(4\pi r^3)/3$, where r is the radius

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Inches Converted to Decimals of a Foot

Inches	Decimals of a Foot	Inches	Decimals of a Foot	Inches	Decimals of a Foot
1/16	0.005	2 1/16	0.172	4 1/16	0.339
1/8	0.010	2 1/8	0.177	4 1/8	0.344
3/16	0.016	2 3/16	0.182	4 3/16	0.349
1/4	0.021	2 1/4	0.188	4 1/4	0.354
5/16	0.026	2 5/16	0.193	4 5/16	0.359
3/8	0.031	2 3/8	0.198	4 3/8	0.365
7/16	0.036	2 7/16	0.203	4 7/16	0.370
1/2	0.042	2 1/2	0.208	4 1/2	0.374
9/16	0.047	2 9/16	0.214	4 9/16	0.380
5/8	0.052	2 5/8	0.219	4 5/8	0.385
11/16	0.057	2 11/16	0.224	4 11/16	0.391
3/4	0.063	2 3/4	0.229	4 3/4	0.396
13/16	0.068	2 13/16	0.234	4 13/16	0.401
7/8	0.073	2 7/8	0.240	4 7/8	0.406
15/16	0.078	2 15/16	0.245	4 15/16	0.411
1	0.083	3	0.250	5	0.417
1 1/16	0.089	3 1/16	0.255	5 1/16	0.422
1 1/8	0.094	3 1/8	0.260	5 1/8	0.427
1 3/16	0.099	3 3/16	0.266	5 3/16	0.432
1 1/4	0.104	3 1/4	0.271	5 1/4	0.438
1 5/16	0.109	3 5/16	0.276	5 5/16	0.443
1 3/8	0.115	3 3/8	0.281	5 3/8	0.448
1 7/16	0.120	3 7/16	0.286	5 7/16	0.453
1 1/2	0.125	3 1/2	0.292	5 1/2	0.458
1 9/16	0.130	3 9/16	0.297	5 9/16	0.464
1 5/8	0.135	3 5/8	0.302	5 5/8	0.469
1 11/16	0.141	3 11/16	0.307	5 11/16	0.474
1 3/4	0.146	3 3/4	0.313	5 3/4	0.479
1 13/16	0.151	3 13/16	0.318	5 13/16	0.484
1 7/8	0.156	3 7/8	0.323	5 7/8	0.490
1 15/16	0.161	3 15/16	0.328	5 15/16	0.495
2	0.167	4	0.333	6	0.500

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Inches	Decimals of a Foot	Inches	Decimals of a Foot	Inches	Decimals of a Foot
6 1/16	0.505	8 1/16	0.672	10 1/16	0.839
6 1/8	0.510	8 1/8	0.677	10 1/8	0.844
6 3/16	0.516	8 3/16	0.682	10 3/16	0.849
6 1/4	0.521	8 1/4	0.688	10 1/4	0.854
6 5/16	0.526	8 5/16	0.693	10 5/16	0.859
6 3/8	0.531	8 3/8	0.698	10 3/8	0.865
6 7/16	0.536	8 7/16	0.703	10 7/16	0.870
6 1/2	0.542	8 1/2	0.708	10 1/2	0.875
6 9/16	0.547	8 9/16	0.714	10 9/16	0.880
6 5/8	0.552	8 5/8	0.719	10 5/8	0.885
6 11/16	0.557	8 11/16	0.724	10 11/16	0.891
6 3/4	0.563	8 3/4	0.729	10 3/4	0.896
6 13/16	0.568	8 13/16	0.734	10 13/16	0.901
6 7/8	0.573	8 7/8	0.740	10 7/8	0.906
6 15/16	0.578	8 15/16	0.745	10 15/16	0.911
7	0.583	9	0.750	11	0.917
7 1/16	0.589	9 1/16	0.755	11 1/16	0.922
7 1/8	0.594	9 1/8	0.760	11 1/8	0.927
7 3/16	0.599	9 3/16	0.766	11 3/16	0.932
7 1/4	0.604	9 1/4	0.771	11 1/4	0.938
7 5/16	0.609	9 5/16	0.776	11 5/16	0.943
7 3/8	0.615	9 3/8	0.781	11 3/8	0.948
7 7/16	0.620	9 7/16	0.786	11 7/16	0.953
7 1/2	0.625	9 1/2	0.792	11 1/2	0.958
7 9/16	0.630	9 9/16	0.797	11 9/16	0.964
7 5/8	0.635	9 5/8	0.802	11 5/8	0.969
7 11/16	0.641	9 11/16	0.807	11 11/16	0.974
7 3/4	0.646	9 3/4	0.813	11 3/4	0.979
7 13/16	0.651	9 13/16	0.818	11 13/16	0.984
7 7/8	0.656	9 7/8	0.823	11 7/8	0.990
7 15/16	0.661	9 15/16	0.828	11 15/16	0.995
8	0.667	10	0.833	12	1.000

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Trade Terms Introduced in This Module

Acute angle: Any angle between 0 degrees and 90 degrees.

Adjacent angles: Angles that have the same vertex and one side in common.

Angle: The shape made by two straight lines coming together at a point. The space between those two lines is measured in degrees.

Area: The surface or amount of space occupied by a two-dimensional object such as a rectangle, circle, or square.

Base: As it relates to triangles, the base is the line forming the bottom of the triangle.

Bisect: To divide something into two parts that are often equal. When an angle is bisected for example, the two resulting angles are equal.

Circle: A closed curved line around a central point. A circle measures 360 degrees.

Circumference: The distance around the curved line that forms the circle.

Cube: A three-dimensional square, with the measurements in all the three dimensions being equal.

Decimal: A part of a number represented by digits to the right of a point, called a decimal point. For example, in the number 1.25, .25 is the decimal portion of the number. In this case, it represents 25% of the whole number 1.

Degree: A unit of measurement for angles. For example, a right angle is 90 degrees, an acute angle is between 0 and 90 degrees, and an obtuse angle is between 90 and 180 degrees.

Denominator: The part of a fraction below the dividing line. For example, the 2 in $\frac{1}{2}$ is the denominator. It is equivalent to the divisor in a long division problem.

Diagonal: Line drawn from one corner of a rectangle or square to the farthest opposite corner.

Diameter: The length of a straight line that crosses from one side of a circle, through the center point, to a point on the opposite side. The diameter is the longest straight line you can draw inside a circle.

Difference: The result of subtracting one number from another. For example, in the problem $8 - 3 = 5$, 5 is the difference between the two numbers.

Digit: Any of the numerical symbols 0 to 9.

Dividend: In a division problem, the number being divided is the dividend.

Divisor: In a division problem, the number that is divided into another number is called the divisor.

Equation: A mathematical statement that indicates the value of two mathematical expressions, such as 2×2 and 1×4, are equal. An equation is written using the equal sign in this manner: $2 \times 2 = 1 \times 4$.

Equilateral triangle: A triangle that has three equal sides and three equal angles.

Equivalent fractions: Fractions having different numerators and denominators but still have equal values, such as the two fractions $\frac{1}{2}$ and $\frac{2}{4}$.

Force: A push or pull on a surface. In this module, force is considered to be the weight of an object or fluid. This is a common approximation.

Formula: A mathematical process used to solve a problem. For example, the formula for finding the area of a rectangle is Side A times Side B = Area, or $A \times B = \text{Area}$.

Fraction: A portion of a whole number represented by two numbers. The upper number of a fraction is known as the numerator and the bottom number is known as the denominator.

Improper fraction: A fraction whose numerator is larger than its denominator. For example, $\frac{5}{4}$ and $\frac{6}{3}$ are improper fractions.

Invert: To reverse the order or position of numbers. In fractions, inverting means to reverse the positions of the numerator and denominator, such that $\frac{3}{4}$ becomes $\frac{4}{3}$. When you are dividing by fractions, one fraction is inverted.

Isosceles triangle: A triangle that has two equal sides and two equal angles.

Joist: Lengths of wood or steel that usually support floors, ceiling, or a roof. Roof joists will be at the same angle as the roof itself, while floor and ceiling joists are usually horizontal.

Loadbearing: Carrying a significant amount of weight and/or providing necessary structural support. A loadbearing wall is typically carrying some portion of the roof weight and cannot be removed without risking structural failure or collapse.

Mass: The quantity of matter present.

Mixed number: A combination of a whole number with a fraction or decimal. Examples of mixed numbers are $3\frac{7}{16}$, 5.75, and $1\frac{1}{4}$.

Negative numbers: Numbers less than zero. For example, –1, –2, and –3 are negative numbers.

Numerator: The part of a fraction above the dividing line. For example, the 1 in ½ is the numerator. It is the equivalent of the dividend in a long division problem.

Obtuse angle: Any angle between 90 degrees and 180 degrees.

Opposite angles: Two angles that are formed by two straight lines crossing. They are always equal.

Perimeter: The distance around the outside of a closed shape, such as a rectangle, circle, square, or any irregular shape.

Pi: A mathematical value of approximately 3.14 (or $^{22}/_{7}$) used to determine the area and circumference of circles. It is sometimes symbolized by π.

Place value: The exact value a digit represents in a whole number, determined by its place within the whole number or by its position relative to the decimal point. In the number 124, the number 2 actually represents 20, since it is in the tens column.

Plane geometry: The mathematical study of two-dimensional (flat) shapes.

Positive numbers: Numbers greater than zero. For example, 1, 2, and 3 are positive numbers. Any number without a negative (–) sign in front of it is considered to be a positive number.

Product: The answer to a multiplication problem. For example, the product of 6×6 is 36.

Quotient: The result of a division problem. For example, when dividing 6 by 2, the quotient is 3.

Radius: The distance from a center point of a circle to any point on the curved line, or half the width (diameter) of a circle.

Rectangle: A four-sided shape with four 90-degree angles. Opposite sides of a rectangle are always parallel and the same length. Adjacent sides are perpendicular and are not equal in length.

Remainder: The amount left over in a division problem. For example, in the problem $34 \div 8$, 8 goes into 34 four times ($8 \times 4 = 32$) with 2 left over; in other words, 2 is the remainder.

Right angle: An angle that measures 90 degrees. The two lines that form a right angle are perpendicular to each other. This is the angle used most in the trades.

Right triangle: A triangle that includes one 90-degree angle.

Scalene triangle: A triangle with sides of unequal lengths.

Solid geometry: The mathematical study of three-dimensional shapes.

Square: (1) A special type of rectangle with four equal sides and four 90-degree angles. (2) The product of a number multiplied by itself. For example, 25 is the square of 5; 16 is the square of 4.

Straight angle: A 180-degree angle or flat line.

Stud: A vertical support inside the wall of a structure to which the wall finish material is attached. The base of a stud rests on a horizontal baseplate, and a horizontal cap plate rests on top of a series of studs.

Sum: The resulting total in an addition problem. For example, in the problem $7 + 8 = 15$, 15 is the sum.

Triangle: A closed shape that has three sides and three angles.

Unit: A definite standard of measure.

Vertex: A point at which two or more lines or curves come together.

Volume: The amount of space contained in a given three-dimensional shape.

Whole numbers: Complete number units without fractions or decimals.

Additional Resources

This module presents thorough resources for task training. The following resource material is suggested for further study.

Applied Construction Math: A Novel Approach, NCCER. 2006. Upper Saddle River, NJ: Prentice Hall.

Mathematics for Carpentry and the Construction Trades, Alfred P. Webster; Kathryn B. Judy. 2001. Upper Saddle River, NJ: Prentice Hall.

Mathematics for the Trades: A Guided Approach, Robert A. Carman; Hal Saunders. 2014. Pearson Learning.

Metric-conversion.org : Metric Conversion Charts and Calculators.

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